Commentary

RESPONSE TO DEHAENE

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Dehaene's model avoids the assumption of numerical processing only by putting it in the tutor instead of in the net. The attempt is ironic, because the units in the net process their inputs arithmetically. Furthermore, treating the different values of *T* as unrelated stimuli leads to a radically nonparsimonious model of what must be learned.

In Dehaene's simulation, the value of a reward is multiplied by an exponentially decaying function of the number of flashes that the subject had to generate after its choice and before obtaining its reward. The computation of the discount factor and its scaling of the value of the reward is carried out by the tutor, but if such processes are causally relevant to the behavior observed, then they have to be located in the brain of the subject. Absent that brain, the effect of the flashes on the value of a food reward is moot. It is not more parsimonious to assume that the brain negates the flash count, exponentiates the negation, and then multiplies the base value of a reward by the result than it is to assume that the brain subtracts. Implementing Dehaene's model in a multiunit neural net is still less parsimonious. Because every unit in a net is assumed to sum signed inputs, only one such unit is required to implement the subtraction that we assume.

Finally, in Dehaene's model, the response to each number of *T* flashes is an independently acquired habit. Nets are universal function approximators (Hornik, Stinchcombe, & White, 1989), which means they will learn an arbitrary mapping from inputs to outputs. Dehaene's net will learn, for example, to choose the standard key after an odd number of flashes and the number-left key after an even number. We know that this is not how numbers function for animal subjects. (See Honig, 1993, for a demonstration that pigeons have great difficulty with odd-even discriminations and Brannon & Terrace, 2000, for a demonstration that monkeys cannot learn to touch arrays of different numerosities in arbitrary nonmonotonic orders but readily learn to touch them in a monotonic numerical order.) Thus, in Dehaene's model, the more numbers there are, the more numerous are the "habits" (S-R relations) that must be acquired. When the values of *I* and *S* are increased, the net must learn a different mapping for each of the *T* values it had previously mastered plus new mappings for the newly introduced values of *T*. Our subjects' rapid adjustments to new values of *I* and *S* are not consistent with the assumption that making this adjustment involves learning a large number of new discriminations. We would welcome, as Dehaene suggests, future demonstrations of numerical subtraction in our number-left paradigm with unrewarded trials; however, given the data available, we view Dehaene's associative explanation unparsimonious and unlikely.

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