Truth Tables and Mathematical Induction

**Question:** Why can we be sure that this procedure exhausts the possible combinations of truth values (for any collection of simple statements $\alpha_1, \ldots, \alpha_n$)?

For any such collection of statements, what we want to do is to construct all the possible n-ary sequences of Ts and Fs.

We will answer our question by means of a technique known as mathematical induction. (See notes on mathematical induction.)

**Base Step.** Suppose $n=1$. Then there is only one statement letter $\alpha_1$ in $\Gamma$. By our logical assumption, $\alpha_1$ is T or F ($2^1 = 1$)

**Induction Step.** Suppose we have constructed all the possible sequences of length $m$. We now want to construct all the possible sequences of length $m+1$.

To do this, it is enough to construct all the possible $m$-ary sequences and then put a T at the end of each one of them, and then to construct all the possible $m$-ary sequences again, this time putting an F at the end of each one of them.

Thus, the number of $(m+1)$-ary sequences is *twice* the number of $m$-ary sequences. Since there are $2^m$ m-ary sequences, there are $2^m \times 2 = 2^{m+1}$ (m+1)-ary sequences.

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