The Notion of Principle: The Case of Counting

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We have proposed that learning to count skillfully is initially directed by the implicit knowledge of some counting principles (e.g., Gelman & Gallistel, 1978; Gelman & Meck, 1983). Here we develop the thesis that this is so because principles both set constraints on the nature of counting procedures and provide clues about relevant learning environments. More generally, the idea is that much of early cognitive development proceeds as a function of some domain-specific principles that define domains, focus attention on domain-relevant inputs, and play a central role in the selection and generation of the class of domain-appropriate behaviors. In this chapter, we expand on the notion of principle, taking up both what it does and what it does not entail. In doing so, we hope to clarify the nature of principled or conceptual competence and its relation to the generation of behavior which is governed by procedural competence.

**INITIAL CONSIDERATIONS**

One can attribute to individuals knowledge of principles without requiring that they be able to articulate those principles. The distinction here is between *implicit* and *explicit* knowledge and is central to discussions in linguistics and psycholinguistics. We honor principles of syntax when listening to and producing utterances that are in the language, as well as when deciding that strings like "Who did John see Mary and?" are not in the language. Yet we are not able to articulate the linguistic principle governing the selection or rejection of acceptable utterances (the present unacceptable string violates the Coordinate Structure Constraint [Ross, 1967]).
Knowledge of the correct principles does not guarantee correct performance. As we will see, principles specify characteristics that a correct performance must possess, but they do not provide recipes for generating a plan for correct performances. Nor do they guarantee correct execution of a plan. As a consequence, a competence-performance distinction is central to our treatment of development. This is one reason Greeno, Riley and Gelman (1984) distinguish between conceptual, procedural and utilization competence.

Although principles do not provide recipes for generating successful procedures under particular circumstances, they do play a fundamental, indispensable role in the synthesis of these procedures. This is because they provide the constraints procedures must reflect if they are to yield acceptable counting behavior. Conceptual competence or principled knowledge is coordinated with the planning- and procedure-generation system that makes up procedural competence and thus helps determine the actual procedures used.

We postulate that some principled understanding precedes skilled counting both because we think the data support the view and because we believe that were there no early competence, children might never learn to count or do arithmetic. The initial competence guides the development of initial procedures. Once the system is on its way, it is inevitable that there will be an interaction between procedural and conceptual competence. We begin our discussion of the relation between conceptual and procedural competence by presenting the case that some principles antedate correct performance and make possible the acquisition of the knowledge required for correct performances. We end with a consideration of a case where the use of a procedure supports the development of further principled understanding. This makes clear that only some counting principles are available to young children before they produce counting behaviors. It also illustrates how conceptual, procedural, and utilization competence can interact to produce new learning (see Baroody & Ginsburg, Carpenter, Sinclair & Sinclair, and Schoenfeld, this volume).

A Concrete Case: The Counting Principles

Counting to assess the cardinal value of a set is governed by five principles. These are: (a) one-one correspondence—every item in a set must be assigned a unique tag; (b) the stable order principle—the tags used must be drawn from a stably ordered list; (c) the cardinal principle—the last tag used in a count has a special status; it represents the cardinal value of the set; (d) the item-indifference (or irrelevance) principle—there are no restrictions on the collection of items that can be counted; and, (e) the order-indifference principle—the order in which items are tagged is irrelevant; a change in the order of the items in the display does not change the consequence of applying the first three principles. Note that the last two principles are principles of permissibility, statements about the wide range of conditions to which the first three “how-to-count” principles can apply.

This or some subset of this list of principles is the one we suggest as a possible candidate for the initial competence children bring to the task of acquiring skill at counting. Note that it does not cover all principles about counting. To do this, it at least needs to be expanded to account, for example, for our knowledge of the relationship between counting and the ordinal number of some object in a display (Botman, 1981; Fuson, Richards & Briars, 1982) or our knowledge that every count number has a successor (Evans & Gelman, 1985). We do not include knowledge of such principles in the initial competence, because we can propose ways in which these principles could be learned—given the assumption of what is already in conceptual competence.

We believe that the function of the early principles is to guide the assemblage of procedures and the development of skill at using these procedures. Other researchers maintain that initially children memorize, by rote and without understanding, various counting behaviors and only eventually induce principles or components of the counting principles (e.g. Briars & Siegler, 1984; Fuson & Hall, 1983). These authors point out that young children lack general skill and that what skills they do have appear only in limited situations. Additionally, they note children do best on tasks that use conventional counting behaviors, for example, counting things in a row as opposed to a circle. In learning theory or S-R terms, the argument is that children’s habits are built up as children are reinforced for copying conventional demonstrations; initial habits are weak, and performance and generalization levels are low. This account of early number skill is developed in detail in Briars and Siegler (1984) and Siegler and Shrager (1984).

We show here that although it is true that young children have limited and variable skill, this is not definitive evidence for the rote-learning position. The fact that initial skills are variable fits well, indeed is predicted, by the position that principles guide but do not guarantee successful performance (see also Wilkinson, 1984). Further, we show that it is difficult to explain why young children learn count sequences at all unless we grant them a structure that guides this learning. We attempt to show that it would be exceedingly difficult for learning about counting to get off the ground in the first place were children not able to take advantage of some counting principles.

Why Postulate Principles?

To Help Solve the Problem of Stimulus Attention and Stimulus Definition. Recent research converges on a rather surprising generalization: babies and very young children attend to numerically relevant information in the environment. For example, 6- to 8-month-old infants match the number of drumbeats they hear (two or three) with the number of items they see in a colored photograph of a haphazard collection of small household objects (Starkey, Spelke, & Gelman, in press). Twelve-month-olds can learn to compare a paired
sequence of displays of one, two, three, or four items on the basis of whether the displays have an equal number of items, one has more items, or one has fewer items (Cooper, Campbell, & Safady, 1985). This is a surprising generalization, because not long ago even preschoolers were said to be unable to achieve true numerical (as opposed to perceptual) representations of sets. Matters pertaining to number were assumed to be too abstract for the young mind (see Gelman, 1982, for further discussion). There has been considerable debate over the way to interpret the fact that babies, toddlers, and preschoolers respond to numerically relevant information. But given the explosion of research on this topic (see Ginsburg, 1983; Sophian, 1984, for many examples), it can be concluded that such information is especially salient to young children. The same cannot be said for chimpanzees (Premack, 1976).

Why the salience? Although it is true that children in our culture encounter a number-rich environment, this will not suffice as an account. Infants are unlikely to have encountered much of this environment, yet they respond on the basis of the number of items in a display or sequence. Further, there is an infinitude of stimuli in the environment to which infants might attend and yet do not. That something is in the environment is no guarantee that it will be attended to. The child’s focus on number words at an early age (Fuson, Richards, & Briars, 1982; Gelman & Gallistel, 1978; Saxe, Guberman, & Gearhart, 1984) is a real puzzle, especially in view of the difficulty that the same age children have learning color terms (e.g., Bartlett, 1977; Bornstein, 1985; Landau & Gleitman, 1985; Rice, 1980). After all, color terms are also omnipresent in the environment. Young children watch Sesame Street; play with colored stacking blocks and colored rings; draw with crayons that produce colors. Surely adults use some of these occasions to tell children the names of the colors they are seeing or working with. It will not do, therefore, to argue that number words are attended to and memorized because they are heard frequently and used in relevant contexts—so are color terms. Yet color terms are not learned early—even though color differences are perceived (Bornstein, 1985).

At the sensory level, it is easy to account for the fact that we attend to some ranges of the light frequency spectrum but not others or that we hear some sounds and not others; the structure of our sense organs plays the controlling role. Similarly, the structure of our knowledge base can determine whether or not we attend to an aspect of our environment. Piaget made this point repeatedly. For example, Inhelder and Piaget (1964) argued that young children fail to notice the orderliness in an ascending series of lengths because they lack the seriation structures needed to assimilate such information. Without taking sides in the debate about whether young children can or cannot seriate lengths, we can endorse the general point. Information of a particular kind is assimilated only if a suitable structure is available to support the assimilation. Indeed, it is often the availability of a structure that makes a stimulus salient; the structure carries with it a characterization of the kind of input it will attend to and process.

We propose that it is the availability of the counting principles that makes children sensitive at an early age to number relevant information, including the counting words. The principles also help the child to sort verbal inputs that resemble each other, thereby allowing the child to match correctly the use of terms to the functions they are meant to serve. Sinclair and Sinclair develop a similar argument in their chapter in this volume.

To illustrate how principles help children match terms and functions, consider the problem of children learning two kinds of words, labels for objects and numerals (numberlog in Gelman & Gallistel’s (1978) terminology) for counting. The theorist who prefers a rote learning account of the ability to learn to count with words will point out that children hear the count words in a variety of contexts and see people pointing to objects, one at a time, as they use the items in the list. The rote learning advocates suggest that these experiences are sufficient for learning to use the count words. The problem with this observation, however, is that the same environment can occur for the names of the objects. Consider a parent “reading” from a picture book: “horse, dog, chair, baby.” Why doesn’t the child who sees an adult pointing to objects in a set while reciting the count list assume that this is a lesson in object naming? The possibility that there may be more frequent demonstrations of counting than recitations of particular words to match particular objects is an inadequate explanation. Color words are recited frequently in response to the ever present set of toys that vary in color; yet children who have mastered from three to six words in an ordered count list and can use at least three of these in a meaningful way (Fuson et al., 1982; Gelman & Gallistel, 1978) have enormous difficulty learning to use even a few color names when given explicit instruction (Bornstein, 1985; Rice, 1980).

So the problem remains. Why is it that children do not seem to think that number words are labels for objects? We suggest that it is the presence or absence of principles that provides the child with clues to what conditions or constraints the use of the terms must meet.

The initial learning of labels for objects benefits from three assumptions children make. First, they assume that it is the object as a whole, rather than any of its parts or its surround, which is its referent (Spelke, 1984). Second, they assume that once an object in a class has been assigned a label, that label applies to other members of the class (Markman & Hutchinson, 1984; Waxman & Gelman, 1985). Finally, they assume that unique objects (e.g., dogs, cats, birds,

1The child’s problem goes beyond determining which class of words support which function. Quine (1960) drew attention to the fact that truly naive learners of a language have no way of knowing whether a label, used in conjunction with a point, refers to the object or parts of that object, what that object can do, the environment surrounding the object, and so forth. Spelke’s assumption that children are constrained first to assume that labels refer to objects goes a long way toward solving their problem regarding labels. See Landau and Gleitman (1985) for a discussion of the possible constraints on verb meaning.
etc.) have unique names; different objects cannot be assigned the same name (Markman, 1984). Contrast these implicit assumptions with the potential use of number words given the counting principles.

One way to satisfy the stable order principle is to use a list to tag items. On the assumption that this principle is available to structure the environment, ordered lists of words should be salient to the child. But this principle by itself cannot tell the child how to use the lists. This depends on the availability of further principles. During counting, the same words must be used with different objects (the abstraction or item-indifference principle) and assigned to but one item at a time (the one-one principle).

In the absence of the stable order principle and item indifference principles, children's assumptions about labels should lead them to think the count words are labels for objects (as indeed they are when used in reference to the symbols 1, 2, 3, etc.). Thus, it is a combined consideration of the stable order and item-relevance principle that leads to the conclusion that number words are not names for objects. So the answer to our question about how it is that children might manage to keep the tasks of vocabulary and number-word learning separate, is that they are guided by principles from each of these domains. The principles in each domain define the domain-relevant items in the environment. The counting principles dictate that the child has to learn a list and provide clues for recognizing what is to be assimilated from the verbal environment. Thus, to account for the failure of the young child to confuse number-relevant with language-relevant inputs, one needs to grant the presence of implicit principles that carry with them constraints on what is to be assimilated from the environment.

Returning to the problem of color terms: inasmuch as these, like nouns, refer to a particular set of stimuli and do not have to be ordered, they should not be confused with the class of count words. If anything, color terms may be confused with labels for things if children do not yet know a label for an object (Landau & Gleitman, 1983; Markman, 1984). Additionally, the learning of color terms may be at risk because the young may assume that attribute terms come in polar opposites (Landau & Gleitman, 1985) and because they have a bias to treat objects integrally (Smith & Kemler, 1978).

In the absence of principled clues about the use of color terms, young children can still learn them by rote. If learning to use count words benefits from principles, and learning to use color terms does not, two predictions follow. First, the onset of the correct use of color terms should be more variable than the onset of the correct use of count words. Second, most children should use some count words correctly before they use color terms correctly. (Interestingly, once the learning starts, the standard demands of serial learning could force a crossover in the two acquisition curves.)

In an initial effort to test these predictions, Rochel Gelman, Eve Clark, and Steve Pinker analyzed the language acquisition transcripts from Adam, Eve, and Sarah (Brown, 1973). All three children were reinforced by the adult they were talking to for correct usage of their first three count words before they were reinforced for their first correct use of one color term. Age of onset of correct counting ranged from 1:7 to 2:6. The age of onset for correct use of a color term ranged from 1:9 to 3:2.

We know of no evidence on how early children treat labels for things differently from count words. Some data on 3- and 4-year-olds comes from Gelman, Meck and Merkin (1986), who asked those children who completed the first order-relevance experiment whether (a) the "baby" (in a row they had been counting) could be 1, 2, and so on; (b) the "chair" could be 1, 2, and so on; and (c) the "baby" could be "the chair," or vice versa. Of 19 3-year-olds and 36 4-year-olds (N = 40/age group) asked this series, 79% and 80%, respectively, answered yes to the first two questions and no to the third. They readily tolerated the reassignment of counting tags while resisting the reassignment of labels for things. Children were not very good at explaining their judgments. A few talked about counting in response to questions on why it was okay to say "2" for both objects but not okay to use the same label for both objects. Explanations about labels included "The baby does not have enough legs," "People sit in chairs," "A baby is a human being and not a chair," "She is not made like a chair" and once even "A chair is furniture." These explanations read like definitions of basic concepts or object classification statements; the very activities presumed to constrain the child to label things exclusively.

Admittedly, these lines of evidence are preliminary. Still, they are consistent with the idea that a fundamental reason for postulating principles is to capture the proposition that the acquisition of early knowledge can be aided by the availability of domain-specific mental structures. This is because children's reactions to their learning environments are mediated by domain-specific principles that specify, to some degree, the relevant inputs for that domain. The general argument is that just as syntax-based principles lead the child to attend to syntax-relevant inputs as opposed to number-relevant inputs when learning syntax, number- and object-relevant constraints on names for things lead children to learn the correct interpretation of word classes and items within them. For the reader who remains skeptical that children work with principles that generate assumptions about whether they interpret language data as relevant to the way they learn to count words, we offer the following summary of a portion of Eve Clark's diary of her son's language acquisition.

I thank Steve Pinker for agreeing to search the computerized language data bank being assembled by Brian McWhinney and Katherine Snow. He searched for the word "color," the phrase "how many," and all instances of counting-words and color-terms—providing both child and adult utterances. Counting and coloring activities occurred at relatively high and quite comparable rates. What distinguished the color sequences was the ubiquitous tendency for the adult to do most of the talking about color. Eve Clark graciously consented to go through the transcripts with me, serving thus as both an expert, and, in a sense, a source of reliability.
D. C.'s first way of marking plurals was to use the word "two," as for example in "I have two shoe" (for "I have shoes"). Having assigned this syntactic function to the term "two," he refused to include it in his counting list despite repeated efforts on the part of a determined adult friend (an engineer, no less) visiting the household. Thus, he counted items with the list "one, three, four, five, six" and did so repeatedly over trials. When, several months later, he discovered the conventional plural morpheme "s," he inserted the missing "two" between "one" and "three"! A term that once had to be applied in a consistent way as a particular syntactic type was now free to enter a count list where it now met the requirement that it not apply to anything in particular.

To say learning is guided by implicit principles is not to say that there are no input requirements. To the contrary, in addition to drawing attention to the relevant environment, the principles characterize the form of the input requirements. They help detect and sort inputs in accord with the functions they can serve. In their absence, the child must rely on rote memorization, and the learning becomes more difficult, variable, and protracted.

To Define a Domain. One need not use the standard count list in a given natural language in order to count. Computers count with 0s and 1s; some cultural groups in Africa count with different hand positions; various groups in New Guinea count while touching body parts in sequence; letters of the Greek and Hebrew alphabets are also number words; hatch marks and the abacus have existed since time immemorial. Nor does one have to touch or point to each item, start counting a given set in the same place each time a count of it is to be rendered, or arrange the items in any particular way. In addition, there are many ways to keep track of the number of items used in a count-on strategy of arithmetic (Fuson, 1982); indeed there are many count strategies that can be used to solve a variety of arithmetic problems (see Carpenter, Moser & Romberg, 1982; Ginsburg, 1983). What renders order to these different acts we call counting? What permits us to say they are all, in fact, examples of behavior in the common domain of counting? The answer must be that it is principles of counting which define the domain. In all these cases, at the very least, the principles of one-one correspondence, stable ordering, cardinality, item-irrelevance, and order irrelevance must hold. To be sure, in the case of complex counting algorithms, we may have to recognize further principles. But these depend on the availability of the ones already listed.

Thus, just as principles of syntax help define the language domain, principles of number help define the domain of counting as well as other numerical skills.

To Account for Variable Performance. Many have drawn attention to the way limited processing abilities in young children can interfere with the display of an ability (e.g. Case, 1984; Shatz, 1978). This is one way to account for the failure of children to reveal their conceptual competence. Our notion of principle leads to another account of how conceptual competence can be masked. In turn, this account makes clear why the same-principle-first theory of counting predicts variable performance—especially for the youngest children. It is developed here much as it was in Greeno et al. (1984), where distinctions are made between conceptual, procedural, and utilization competence.

In the Greeno model of counting, conceptual competence characterizes knowledge of the counting principles; utilization competence deals with the ability to assess the performance requirements of a particular task in light of the constraints imposed by conceptual competence; and procedural competence deals with planning knowledge about the relations between actions and goals and the ability to make use of constraints. More generally, procedural competence plans the course of action, which satisfies the constraints imposed on it by conceptual and utilization competence. Successful generation of a performance plan, then, depends on the coordinated applications of conceptual, procedural, and utilization competence.

In distinguishing between procedural and conceptual competence, we introduce the idea that the counting principles represent a set of abstract constraints the mind places on procedures. They place constraints on procedures in the sense that the counting procedures must satisfy them if they are to be recognized as instances of counting. Principles, however, do not dictate how to put the principles into practice in any setting. Instead the counting principles provide the specifications that define the classes of acceptable procedures and number-specific goal structures, much as do rewrite rules in linguistics. For example, in the Greeno et al. (1984) computer model, the schemata MATCH and KEEP-EQUAL-INCREASE specify the properties a procedure must exhibit if it is to be consistent with the one-one principle. Since the MATCH schemata takes two sets as arguments, the sets can be (1) some particular items to count and (2) the words in the conventional count list. The sets could also be words, abstract thoughts, or the like on the one hand, and the alphabet or hatch marks on the other. KEEP-EQUAL-INCREASE requires that the two sets remain equal throughout a count and indicates a general way of keeping the sets equal; the idea is that two initially empty sets, remain equal if each is increased by a single member. These schemata together capture the fact that the child will have to generate procedures to partition to-be-counted and already counted items and coordinate the use of count words and the partitioning of items. But they do not dictate how this will happen. Their characterization in terms of abstract action schemata makes it possible to detail the constraints involved in their use and formally describe the class of acceptable procedures. It is the work of procedural competence to generate the particular instances of this class.

Although principles do not themselves generate counting procedures, they do play an indispensable role in their generation, this because they are referred to by the planning or executive part of the procedural competence system, the part that
must produce and monitor the production of competent or effective behaviors. For a child to succeed, procedural competence must produce a competent plan of action. Because the quality of the planner sets the limits on procedural competence, children can fail a task because of a deficit in their procedural competence.

The generation of procedures also requires a utilization competence, the ability to assess a particular setting in terms of the task demands relevant to putting a plan of action into effect. The items in front of a child could be in a circle, in which case the task introduces a counting-relevant feature, that is, the beginning or end of the set must be clearly marked in order to avoid double-counting items and violating the one-one principle. Since children may know that they are not supposed to double-count, yet lack the resources to deal with the problem, a distinction should be drawn between these abilities. To recognize the problem, children have to appreciate the difference between displays that have a clear beginning and end, but this is a problem in perception and is not unique to counting. Deficiencies in the detection or construction of perceptual groupings will influence counting whether or not children possess the requisite conceptual competence: Hence, the Greeno et al. (1984) argument is that these are best represented as deficiencies in utilization competence. Similarly, tendencies to misinterpret instructions, memory problems, and the like will influence the quality of a plan of action because of the limits they place on the child's utilization skill at the moment.

But even if the child has the wherewithal to assess all that matters in a task setting and correctly interpret instructions, there is no guarantee that success will follow. In the case at hand, the child also has to generate an acceptable counting procedure. This might involve moving items; it might also involve putting a piece of paper on an item. Neither of these skills is counting-specific: they could be used to improve the perception and memory of other material as well. Still, a plan which fails to include them or other components which reflect the needs of a setting would be inadequate. For accurate performance to occur, children have to refer their assessment of the task requirements to the planner, which has to coordinate these with the constraints given to it by its conceptual competence.

The foregoing analysis leads to a renewed defense of the "competence-performance" distinction in developmental accounts. Since principles do not spell out how conceptual competence will be put into practice, it follows that there must be a distinction between "competence" and "performance." Our use above of the terms "principled" or "conceptual competence" to substitute for what is standardly taken as "competence" in the phrase competence-performance, this being the underlying rules of knowledge of a domain. However, we characterize the competence more broadly, in terms of the requirements a plan must meet. It is not a passive structure. It offers guidelines for connecting knowledge constraints to plans of action.

Our discussion of what influences performance is different from ones that consider limits on memory, attentional capacities, or processing space (e.g.

Case, 1984; Chomsky, 1957). We also consider the child's ability to plan actions consistent with the constraints set down by the principles. The principles do not guarantee that the child will use the relevant goals; this must be part of the planning activity. Faulty planning could cause a child to fail. Further, a consideration of planning competence requires considering the separate ability to assess task settings and relate them to these constraints. Making a distinction between procedural and utilization competence allows us to begin to classify in a systematic way the different kinds of variables that will influence the selection and production of actions and the consequent performance. We begin this classification in the Evidence section of this chapter.

To Begin to Explain Learning. Since principles set constraints on the acceptability of a chosen procedure, they are a potential source of feedback to children when they are learning to count or are trying to solve a problem that requires a novel behavior. Procedures used can be monitored and checked for violations of the constraints. If there is a discrepancy between the constraints' requirements and an output, the child can obtain information about negative efforts and then reject or alter them to conform more closely to the requirements of the constraints. If there is no discrepancy, the child can use the procedure again and, thus, develop skill as a function of practice. In this sense, then, principles both guide and help determine the acquisition of skill. Procedures that honor the requisite constraints can be used again and gain strength.

The foregoing might lead one to conclude that we give the very young child the conscious ability to monitor cognitive processes and capacities. We do not and see no need to do so. We understand, however, why others might reach this conclusion. The reason is that discussion of the development of metacognitive skills often deals with the ability to monitor one's cognitive processes as if they were on the same plane as the ability to think about the nature of the contents of one's memory. To talk about the ability to monitor is to talk about the regular state of affairs for any case of active learning. Following Piaget (1976, 1978), the argument is that any learning governed by a structure will self-regulate. To capture the tendency of children to self-correct their language errors (Clark & Clark, 1977), their motor errors (Bruner, 1973), their counting errors (Gelman & Gallistel, 1978), or their search errors (DeLoache, 1984), one can use Piaget's phrase "autonomous regulation" without ever referring to matters of consciousness (Piaget, 1976, 1978). Over and over again we find that the development of conscious knowledge of one's implicit knowledge and abilities develops relatively late (Brown, Bransford, Ferrara, & Campione, 1983). But, likewise, there is evidence that very young children monitor, rehearse, and self-correct their performances (Brown et al., DeLoache, 1984; Siegler & Shrager, 1984).
One can recognize the generality of monitoring tendencies without reaching the false conclusion that the very young must, therefore, be metacognitive (see Brown et al., 1983 for more on this issue). Whether or not they possess metacognitive skills, young children do monitor and correct their performances in a host of domains. They could not if there were no representation against which to do so and, of course, if they were not intrinsically motivated to do so, at least some of the time. That they do have representations and are actively engaged in using them to learn is part of what we mean when we say young children use their principles in the pursuit of skill at counting. Additionally, we mean that procedural competence makes it possible for children to do this since it provides a mechanism for generating feedback internally.

As we will see, when the procedures a child puts into practice reflect the constraints of conceptual competence, the output of these procedures can serve as evidence for the learning of new principles. The development of this argument is facilitated by the consideration of a concrete case. Since this case emerged in one set of our studies of how failures in utilization and procedural competence mask conceptual competence, we turn next to the matter of evidence. We will end with a discussion of how young children could learn about the ordinal principle of counting.

EVIDENCE

Conflicting Interpretations

Gelman (1982) and Greeno et al. (1984) cite several lines of evidence for the view that young children possess implicit knowledge of the counting principles. However, a common feature of this evidence is that children are not perfect and the younger they are the more they err. That young children's performance is variable within and across tasks is highlighted by critics of the idea that principles guide the acquisition of procedural skill. The view is that there is little reason to grant principled understanding if there is a variety of conditions under which young children are not competent (Briars & Siegler, 1984).

Associations-first/principles-later accounts of early number abilities handle performance variability by simply granting to young children fewer and weaker rotely learned components of the counting procedure than to older children. It should now be clear that variable performance can also be handled by the some-principles-first view. In fact, it is predicted, given the Greeno et al. (1984) analysis of competence. By way of a brief reminder, even if young children have complete conceptual competence, there is no guarantee they will display it in their behavior. This is because conceptual competence does not consist of recipes for procedures. It defines the class of acceptable procedures, but these must be generated by procedural competence. Deficits in general problem solving and planning abilities could lead to a failure to use conceptual competence. So could deficits in utilization competence. Finally, problems could arise because children have had little experience coordinating conceptual, procedural, and utilization competencies.

Thus far, then, the two classes of competing theories can account for the same data. One advantage to our analysis is that it provides a tool with which to analyze sources of variability. Some variability may reflect a lack of principled understanding. But this does not mean that all variability should be interpreted in this way. A consideration of the tasks used in the studies at issue points to problems children had with utilization and procedural competence. In the next part of this section we propose that utilization and procedural competence problems masked conceptual competence in several studies cited against our position. We then present studies designed to test these hypotheses. The section concludes with a summary of recent studies of the constrained counting ("doesn't matter") task developed by Gelman and Gallistel (1978) because the principle-first and skill-first theories make opposite predictions regarding what 3- and 4-year-old children could do on this task.

The One-One Principle: Error-detection Reconsidered

In their assessments of preschoolers' understanding of the one-one principle, both Briars and Siegler (1984) and Gelman and Meck (1983) included correct but unconventional trials as well as standard-correct and standard-error trials. For example, in both studies, children watched a puppet count a row of alternating red and blue (or green) chips by first counting the red ones and then the blue or green ones. Gelman and Meck labeled these trials pseudoerrors; Briars and Siegler called them unusual correct trials. Gelman and Meck reported an average of 96% correct for their 4-year-old group. The comparable figures from the latter study were 65% and 53%, respectively, for 4- and 5-year-olds. An even greater discrepancy was reported for the 3-year-olds—95% in Gelman and Meck versus 35% in Briars and Siegler. Why the differences on these kinds of trials, given similar results with remaining trial types? We think it is because these trials are ambiguous. Since ambiguity makes demands on one's ability to decide which alternative the experimenter had in mind, it also makes demands on one's utilization competence.

Children and adults alike usually count systematically from left to right or vice versa. Although we need not, doing so helps us keep track of items and honor other constraints placed on procedural competence. Hence, the procedure is likely to be reinforced and become conventional. When we are asked whether a procedure is right or not, there is then the problem of deciding whether to judge on the basis of
what is done typically or whether to use some other criterion. If we choose the former, we should say an unconventional trial is wrong. If we set aside the matter of convention and are competent to judge on the basis of principled considerations we might say it is right. Young children typically choose the conventional road unless given a clue to do otherwise (Braine & Rumain, 1983). Therefore, in the Briars and Siegler study, the children may have assumed they were to treat any deviation from standard counting as wrong—especially since they were told that the puppet “knows his numbers.” They were also given repeated experience counting a row of objects before every block of testing and hence may have been set to focus on the conventional (see Gelman, Meck & Merkin: Study 4, 1986, for evidence that this happens). Children in the Gelman and Meck (1983) study may have been more flexible because they were told the puppet was “just learning to count” and did not themselves repeatedly count linear arrays throughout the experiment.

Inspection of the Gelman and Meck (1983) transcripts disclosed another hint about the discrepancy and suggests a way to pursue our hypothesis. Children were asked to explain their judgments and were tested in a very interactive mode, which means some trials were tested more than once. This does not seem to be the case for the Briars and Siegler study (personal communication). As a consequence, without planning to, we may have given children a hint about how to approach the difference between conventional and unconventional trials (Vygotsky, 1978).

These arguments focus on the task demands children had to deal with in this seemingly straightforward situation and point to utilization competence as the source of difficulty. They also highlight the potential role of conversational skill and social variables in a child’s assessment of ambiguity. If the children did have such problems, Briars’ and Siegler’s (1984) interpretation of their data as contradicting our position is less compelling.

To assess the above hypothesis, we ran 4- and 5-year-olds (N = 10/group) using an interactive testing procedure in an error detection study demonstrating four kinds of one-one correspondence trials: Correct, Error, Pseudoerror, and Compensation Error. The last involved the puppet’s making two compensatory errors, e.g. first double-counting one item and then skipping another. Thus, although the puppet used the same number of tags as there were items, he did not keep the increases equal in both already used subsets.

To do well throughout this task, a child has to infer a procedure that could have generated the display and assess it in terms of the constraints of the counting principles. Thus, clues about how to interpret novel arrays cannot affect success levels unless the children possess the relevant procedural and conceptual competence. Indeed, if the children generally possess these competencies, the effects of an interactive procedure should be limited to those situations that are ambiguous, in this case the Pseudoerror trials. To find out if the effect was so localized,
alyses were done twice: once with children’s immediate responses and once with their best responses on a trial. In the latter case, a child’s immediate response was scored again unless an initial error was followed by a correct response that was justified, for example, “I think it was another one of those okay but silly” trials (in response to a pseudoerror). A child who explained his response could hardly have responded randomly or switched his answer because of a perceived challenge to his initial reply.

After the error detection phase, children were probed in further ways, including a request to assume the role of the experimenter and generate correct and incorrect trials.

Each child was tested with sets of five and seven trinkets, in that order. There were eight trials per set size, two for each of the trial types. On Correct trials, the puppet counted a linear array from beginning to end. The One-one correspondence error trials involved either skipping or double-counting an item. In the Pseudoerror trials, the puppet either started counting in the middle of the array and then returned to the beginning to count the remaining items or skipped an item in the middle of the array and returned to count it last. In the Compensatory error trials, the final tag used in the count list was the same as the cardinal value of the set; however, two errors that cancelled each other were made during each trial—one on trial one item was skipped and a subsequent one was double-counted, and on the other trial the puppet counted a nonexistent item on the table and then skipped another item in the array. Instructions were as given in Gelman and Meck (1983), and ended with the children being told that they had to wait until the puppet had finished before deciding whether the trial was correct or not. An attempt was made to elicit an explanation for each judgment. Where it seemed a child’s interest was lagging, the testing was broken up over two days.

Error Detection Results. Figure 2.1 shows the overall percentage of correct responses for both the Immediate and Best response analyses. As in previous experiments, children were flawless in judging Correct trials on their first encounter with them. Likewise, they did very well on the remaining 12 error trials. Four-year-olds got a mean of 9.3 of their immediate responses correct; 5-year-olds scored a mean of 10.5 correct. On the best-trial analysis, the respective means for the two age groups were 11 and 11.4. Since so many children achieved perfect scores, we could not do an overall analysis of variance.

The age difference for the Immediate-response data from the three kinds of error trials is reliable as assessed by a 1-tail independent t-test (t(18) = 2.78; p = .006). The Best-response difference is not statistically significant (t(18) = 1.447; p = .08). For the younger group, the differences between Immediate-response and Best-response scores on their error trials were significant for both set sizes (correlated r’s for 1-tail tests with df = 9 were 2.21 and 2.24, p = .03 and .01, for set sizes of 5 and 7, respectively). These differences were not
group even produced order-irrelevant correct trials, a kind of trial they had not seen during testing. Five children (three and two, in the young and old groups) introduced errors in the count list, another novel error. Otherwise error trials were like the skip, double-count, and compensation (rare) trials the children had seen.

Summary. We proposed that utilization factors contributed to the discrepancy between the results reported by Briars and Siegler (1984) and by Gelman and Meck (1983). The fact that pseudoerror trials are ambiguous led us to predict that children would benefit more from a second chance with a pseudoerror than with a more standard error or correct trial. The results support the proposal that variability in performance may reflect utilization as opposed to conceptual competence deficits. A similar conclusion holds for the task used by Baroody (1984) to assess children's understanding of the order-irrelevance principle. As we will see, subtle variations in question types can lead children to do either very well or very poorly.

Order-Irrelevance and Cardinality: A Follow-up of Baroody (1984)

Inasmuch as children in the foregoing study judged pseudoerror trials correct and sometimes gave accounts of why, one might conclude they understood the order-irrelevance principle. Baroody (1984) cautions against this assumption, preferring instead to conclude that children can be indifferent to the order in which items in an array are tagged and yet not assume that two different orders yield the same cardinal value. If this is so, Baroody is correct in saying that children do not understand the order-irrelevance principle.

Baroody's conclusion is based on the following task. Children were shown a row of eight items and asked first to count left to right and then to indicate the cardinal value of the set. Then, while the experimenter pointed to the rightmost item, they were asked, "Could you make this number one?" Children typically answered yes. Then the experimenter covered the array and said, "We got N [where N stood for the child's cardinal response] by counting this way [Experimenter points to initial way]. What do you think we would get counting the other way?" The large majority of 5-year-olds (representing a rural sample) responded with some value other than N. Most 6-year-olds were correct.

Gelman et al. (1986, Study 1) replicated the Baroody result with a group (N = 12) of 4-year-old urban children. (An initial pilot study with 5-year-olds yielded too few errors). They also compared this group of children with two others of twelve each on Count and Altered-Question.

Gelman et al. hypothesized that the younger children took the Baroody (1984) instructions for the second half of the trial as a challenge to their first answer,
Findings. The results are straightforward. Only one child (of 12) in the Baroody condition was correct; all others changed the value of the cardinal number they gave the second time. Half the children in the Count group changed their answer; half did not. In contrast, there was little tendency for those in the Altered Question group to change their answers. Ten out of twelve did not, and most gave lucid accounts of why not. Consider G. J. (53 months), who said there were eight to start and then said there would be eight should there be a second count.

Because that way [the first way] there was 8 and there’s no way you can try and change numbers. (Why not?) Watch. [S counts.] (How come you knew there would be 8?) Because I knew that . . . (But you knew even before you counted them. How did you? . . . How could you change the number? What would you have to do?) You would have to put more things on the table or take things away.

Our interpretation of the effects of language in the Baroody condition is buttressed by the fact that children in all three conditions did well on their subsequent tasks. Nine of the children in the Baroody replication condition passed the Trick trial; so did 10 in each of the other two conditions. Finally, all groups did well on the Error-detection trials. The average number correct (immediate) responses on the 12 error trials were 10.3, 11.0 and 10.75 for the Baroody, Count, and Altered-Question groups. Although children did find the one-one trials the hardest, they got 82% (immediate response) correct. And no fewer than seven children in each group got five out of six of their trials like this correct (chance $p \leq .03$ by a binomial expansion). Further, when in this phase of the experiment, 28 of the 36 subjects made explicit reference at least once to the fact that the resulting count sequence, the cardinal number, or both were wrong on such trials. In fact, 21 of these children corrected the puppet. Finally, it turns out that the experimenter sometimes challenged children and suggested that the trial was right as long as the puppet repeated the last tag used in the second count list. For example, on a set size of 11, the experimenter said, “But he counted six and he said the six.” One child came back with, “No. He had to go 1, 2, 3, 4, 5, 6, 7, and that’s seven.” Thirteen of the 16 who were challenged responded similarly.

Discussion. The results of this experiment lead to the conclusion that children may fail tasks designed to tap their understanding of the order-irrelevance and cardinal principles because of problems in the domain of utilization competence. It is well known that young children are sensitive to variations in the social context in a way that older children are not. Whether one talks about their being more dependent on a supporting social context, less able to stand back and reinterpret communication messages, or more likely to misinterpret instructions, the theme is clear. This is a general issue and not restricted to the domain of
The generality of the phenomenon strengthens our argument that utilization factors influence the generation and assessment of procedures. One might accept our conclusion regarding 4-year-olds in the Gelman et al. (1986) study and still maintain that matters are different for younger children. For this reason Gelman et al. ran a study of 3-year-olds (Study 2), comparing children in the Baroody condition and those in an Experimental condition. The latter combined features of the Count and Altered-Question conditions used earlier with the 4-year-olds. This was done on the assumption that younger children would be even more in need of instructions that induced confidence and did not challenge them. Two to three months after testing in these conditions, a Trick and Error-detection phase was administered in both groups.

As before, children in the Baroody condition changed their answers more often than those in the Experimental group. Likewise, both groups did well on the Trick and Error-detection tasks. Thus, we can conclude that the argument outlined earlier applies as well for a group of children with an average age of three and a half years.

It is not just utilization factors that are highlighted by these findings. Consider the fact that children did well on both the trick trials and the one-one error-detection trials. Fuson and Hall (1983) have argued that young children’s tendency to repeat the last tag they recite when counting is consistent with their conclusion that young children lack the cardinal principle, for they can do this while following the simple rule “Repeat the last number you heard.” If so, children could not have succeeded on the follow-up tasks. Instead they should have said the puppet who did repeat the last tag he used was correct on the trick and the one-one error trials. Since they did not, they must have used cardinal representations built up on previous trials and compared the values they expected with those rendered by the puppet. We conclude that they made use of the cardinal principle—as one hopes they did, given how well they performed in the Altered Question condition. (Parenthetically, this means these tasks can be used to sort children on standard counting tasks into those who repeat the last tag they use—simple repeaters—and those who are applying the cardinal principle.)

If young children do have implicit knowledge of the cardinal principle, why would they ever resort simply to repeating the last tag they hear or say and ignore whether a puppet’s or their own application of the counting procedure was correct? (Fuson & Hall, 1983). Perhaps they would if task conditions suggested they should keep separate their tagging and cardinal response goals. This could happen if children were first asked to count and allowed to do so and then asked to answer the “How many?” question—just as they were in Ginsburg and Russell (1981) and the study cited by Fuson and Hall (1983). In the Gelman et al. (in press) tasks, children were encouraged to wait before answering until all components of the counting performance were complete. In other words, the task setting encouraged their integration of the various goals and subgoals involved in the assignment of the cardinal value of a set. If our account is correct, then the child’s problem lies in the realms of both procedural and utilization competence. A failure to maintain a number-relevant goal could indicate a fragile coupling between conceptual and procedural competence, a limit on the side of procedural competence, or a failure to render a correct interpretation of the task. These are nontrivial developmental problems and may even reflect what Baroody and Ginsburg (this volume) call “weak schemata.” But they cannot be ascribed entirely to the lack of a principled understanding—not in view of the findings of the present experiment.

Order-Irrelevance: Constrained Counting

Return to the Briars and Siegler (1984) account of 3- and 4-year-olds’ trouble with pseudoeerror trials. This is that the younger the children, the more likely they know but a few conventional counting acts, ones they have had ample opportunity to observe and learn by rote. If so children of this age should not be able to generate novel ways of counting, which is exactly what children must do in order to pass the Gelman and Gallistel (1978) constrained counting task. In this task, children cannot succeed if they think linear arrays can be counted only from one end to the other. The reader is referred to Fig. 2.2 as an aid to the following description of why this is so.

After a child first counted a five-item array, she was asked to show a puppet some tricks. The first trick was to count while making the object in the second position (the “baby”) be the “one.” If the child succeeded on this trial, she was then asked to make the baby be the “two,” the “three,” the “four,” the “five,” and finally the “six.” Each row of Fig. 2.2 schematizes a heterogeneous set of trinkets. The X inside a circle indicates to which object the labeling constraint applies. The order in which objects were tagged, as well as which numeral was used as a tag, is illustrated by the flow of arrows. The value in the right hand column indicates which tag the child was asked to use to designate the object marked with an X. To be scored as correct on a problem

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4Baroody and Ginsburg (personal communication) suggest that less mature children might lack the order-irrelevance principle and think that the set size of a display changes as a function of the order in which they tag items. We have no data to rule out this possibility. However, when so few children fail a test, perhaps an appeal to measurement error is appropriate. Further, we draw attention to the fact that tasks like ours and Baroody’s require children to reason about the consequence of a previous act vis-à-vis future acts. Following Braine and Rumain (1983) it seems reasonable that success on this class of tasks requires an analytic mode of thought. Therefore, children who do succeed can be said to have accessed implicit knowledge about counting. In other words, they may have demonstrated a beginning explicit understanding of the order-irrelevance principle, a conjecture supported by their ability both to explain their predictions and to reject a request to make a particular object the X + 1th tag in the “doesn’t matter” experiments described later.
Smith and Greeno (1984) have shown that all but one of the solutions in Fig. 2.2 can be modeled by upgrading the procedural and utilization components of the Greeno et al. (1984) computer model. The conceptual competence is left intact, thereby providing yet another demonstration that the constrained counting task places special demands on utilization and procedural competence, ones that can mask conceptual competence.

LEARNING NEW PRINCIPLES:
OR WHY CONCEPTUAL, PROCEDURAL, AND
UTILIZATION COMPETENCE INTERACT

All solutions but that shown in line 4 of Fig. 2.2 are examples of what Gelman et al. (1986) called Skip-around solutions. The exception, that for the X = 4 trial, is an example of the Correspondence-Capitalize solutions used by almost all children. These, as well as the Correspondence-Create solutions shown in Figs. 2.3 and 2.4, cannot be generated by the Smith and Greeno (1984) version of the counting model for a reason that is especially interesting. To do so, the model would have to add to conceptual competence schemata representing the ordinal counting principle, something we are not willing to place in the set of initial

FIG. 2.2. Schematic representation of the first half of the Gelman and Gallistel (1978) “doesn’t matter” test of the order-irrelevance principle.

block, a child had to get all but the last trial right. The N + 1 responses were scored separately.

Children have to count in nonstandard ways in order to pass this task. Only 12% and 44% of the 3- and 4-year-olds did pass. Does this mean that the children lacked the ability to generate novel counting solutions? The answer is no. Gelman et al. (1986; Studies 3 and 4) provide evidence that the utilization and procedural demands of a five-item display interfered with 3- and 4-year-olds’ ability to generate novel and strategic solutions. When the same aged children started with a three-item problem of the same form, they did very well on both the regular and N + 1 trials. They also were able to transfer to four-item and five-item problems.

FIG. 2.3. Examples of strategies children used in the Gelman, Meck and Merkin “doesn’t matter” 3-item problems. Dotted lines show the way children moved items.
principles young children bring to the task of learning to count. Otherwise, the children must learn a principle or component of a principle not covered in our account of initial conceptual competence. The Greeno et al. (1984) model fails to generate these strategies then because it lacks the additional requisite conceptual competence.

Why assume the children had this additional competence? How could they have learned it? The first question is answered by a more careful look at the strategies the children invented.

The children who used the Correspondence-Capitalize strategies, did so almost invariably after using a Skip-around solution. They did this only on those trials where the ordinal position of the target item in the display and the particular count word it was supposed to be tagged with would be in correspondence if the child started to count at one end or the other. Thus, after using a Skip solution for a request to make $X = 1$ when $X$ was in the second position, children typically started at one end of the array and counted to the other end when asked to make $X = 2$. Rather than continue to use the Skip strategy, they moved to a smooth left-to-right (or vice versa) count. More telling were those cases where, for a five-
strains of conceptual competence to produce a procedure that in turn sets up a potential learning experience. In other words, what seemed at first a procedural nicety turns out to be much more. Competent procedures, that is, ones that are tied to conceptual competence, generate opportunities to learn principles the child does not know. In the present case, the procedure for insuring a proper application of the counting principles provides the basis for the realization that the numbers may also be used to represent position in a nonnumerical ordering—this could then be the beginning of the ordinal use of number words.

In this discussion of learning, procedural competence plays a central role. It generates experiences that make it possible to enlarge the domain of conceptual competence. But note how closely tied procedural competence must be to some conceptual competence for our account to go through. If young children did not make use of the stable order principle, they would not be in a position to recognize the correspondence between numerical and linear order. Further, if there were no reason for children to use procedures that allow this—if the procedures were not generated in response to the constraints of the counting principles already available to the child—children could not contrast the cases where order is irrelevant and where it is relevant. By granting the child certain basic principles for obtaining numerical representations, we also provide an account of how new representational principles may be acquired. If we deny children any principles at all, we deny them the very tools they need to assimilate structured information. For they have no representation against which to assess whether they have encountered something that is related to what they already know. It is for this reason that we conclude that conceptual competence must interact with utilization and procedural competence if learning is to continue.

CONCLUSION

In this chapter we developed the thesis that some counting principles constitute the conceptual competence young children use to select from their environment numerically relevant information, to build procedures for generating count sequences, and to gain skill at counting. We showed that the fact that young children are variable in their ability to succeed on counting tasks can be explained by both a principle-first and principle-after account of early numerical skill. In doing so we developed the theme that counting competence is made up of conceptual, procedural, and utilization competence. A major consequence of this analysis of competence is that it provides a way of showing how procedural competence can lead to the development of new principles of conceptual competence.

Ours is a theory of how number knowledge can start to expand in the domain of counting. Because it depends on the assumption that children start with some components of conceptual competence, one might conclude it does not apply to the learning of more complex kinds of arithmetic—let alone higher mathematics. But we think there is a generalization to be drawn. In our account of how new principled knowledge is acquired, procedural competence leads to learning if it reflects some existing conceptual understanding. When procedures honor the constraints of what is understood, a major condition for learning is met. This is because the procedures, in response to the constraints of a setting, sometimes create environments that present structured inputs, inputs which serve as fuel for the induction of a concept which is related to ones already known. If we are correct, then one problem facing mathematics instructors is to figure out how to get students to use procedures that do reflect at least a bit of conceptual competence on their part. Carpenter’s discussion of arithmetic and Schoenfeld’s of geometry (this volume), are examples of this approach to pedagogy.

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