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Discussion

Commentary on Le Corre & Carey

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The experiments reported by Le Corre and Carey were designed to test a form of the analog mapping hypothesis, but not to test the hypothesis advanced by Gallistel and Gelman (1992). Our hypothesis is that very early in the prolonged process of learning verbal counting children recognize[2] that the structure and function of verbal counting are the same as the structure and function of their pre-verbal counting system. Both processes honor the one-one, stable order, and cardinal principles, and both processes deliver symbols that are subject to arithmetic processing. Gelman and her coauthors (Gelman, 1998; Gelman & Lucariello, 2002; Gelman & Williams, 1998) have argued that this recognition of common structure and function is mediated by a more general process that she calls structure mapping, which plays an important role in many other aspects of cognitive development. The Gallistel and Gelman (1992) hypothesis is that it is the early recognition that there is a mapping of both structure and function that drives the long, slow process of learning the bidirectional maps – the mapping from the mental magnitudes to the corresponding count words and the mapping from the count words to the corresponding mental magnitudes. The process of learning these two mappings never goes to completion in that count words like 'million', 'billion', and 'trillion' probably do not map to useable mental magnitudes in most adults.

On our hypothesis, counting is a numerically meaningful activity even in children who are very bad counters. They already understand that counting yields a word that represents cardinality and that words representing cardinality are principal players in talk about arithmetic operations, such as addition and subtraction. Thus, we take

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[2] 'Recognition' is not to be taken to mean conscious recognition.

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strong exception to claims made by Le Corre and Carey in their introduction, such as "...while [three year olds' counting up to ten] has the form of the adult count list, this early count list is numerically meaningless." Large portions of Gelman and Gallistel (1978) were devoted to reviewing experimental findings showing that counting at this early age is not numerically meaningless. Perhaps the single most telling finding is that when the cardinality of a set that is for one reason or another the focus of the child's attention is brought into question, young three-year-olds, and even older two-year olds, resort to counting (Gelman, 1972, 1993; Zur & Gelman, 2004; see Gelman, 2006, for review) . The counting that appears when the cardinality of a set is brought into question is better than is observed when the same children are asked simply to count a set of objects (Gelman, 1993). Moreover, these very young children seem already to recognize that abstract arithmetic principles, such as that subtraction and addition are inverse operations, apply to the numbers represented by the count words (Bisanz, Sherman, Rasmussen, & Ho, 2005; Groen & Resnick, 1977; Zur & Gelman, 2004). These findings are not easily reconciled with the assertion that counting at these early ages is numerically meaningless, nor with terminology such as "CP-knowers," (and by implication non-knowers), which implies that many children in their late preschool years do not yet know the counting principles.

The form of the analog mapping hypothesis that Le Corre and Carey test is that children learn the meaning of (at least the first four or five) count words only by learning a mapping from the corresponding mental magnitude to that count word. On this hypothesis, if I understand them correctly, the child does not know what a count word means until it knows how to map from the appropriate mental magnitude to the word. In their version of the analog-mapping hypothesis, it is assumed that the child must learn the mappings before it learns the counting principles. The implicit hypothesis appears to be that the counting principles could somehow be acquired by induction from the learned mappings, although what the inductive principle might be is not specified. Throughout their introduction, they take it for granted that counting principles are acquired only after mappings of some kind have been established. The only question they entertain is mappings to or from what (mental magnitudes, symbols in working or long-term memory, the system for representing the meaning of natural language quantifiers, or some combination thereof). In their framing of the questions, our hypothesis, which is that the principles that govern verbal counting are recognized by analogy to the principles that govern non-verbal counting before any mappings are learned, is not on the table.

Their experimental approach is dictated by their framing of the question. Their fast-cards task is used to assess the course of the development of the mapping from visual arrays with varying numbers of items portrayed to the appropriate number word, under conditions where counting (possibly even non-verbal counting, but certainly verbal counting) is not really possible (because of the short display times) and is discouraged when it is attempted. They correlate the ability of children to give roughly appropriate number words for arrays up to 10 with whether the same children attain a certain level of performance in their give-a-number test. The attainment of that level of performance in the give-a-number task is taken to indicate acquisition of the counting principles (the attainment of "CP-knower" status).

Children were judged to know the counting principles when they gave the requested number of objects for each number up to at least 6 at least 2/3 of the time and gave that same number no more than 1/3 of the time when asked for another number. From my perspective, this is a test of children's skill at using, or inclination to use, whatever mastery of verbal counting they have attained, in order to honor the experimenter's request. I have no idea why many 4-year olds do so poorly on this task. I do take this as evidence that they are not yet good counters or at least that they cannot deploy their skill in a task that would be easy for an adult. But it is not evidence that they do not understand the counting principles. When even younger children are allowed or encouraged to count in a similar task, they do count and they do much better at correctly stating the number of items in the array (Gelman, 1993). Children this age and younger already regard the verbal representation of cardinality achieved by counting as taking precedence over an estimate they have made by non-verbal counting (Zur & Gelman, 2004). This is evidence that they do understand both what the principles governing verbal counting are and what the purpose of it is. Their problem appears to be not with the principles but rather with the practice, and even that problem appears to be strongly task dependent. For a discussion of why it takes a long time to learn to put the counting principles into effective practice, see Gelman and Greeno (1989).

I turn now to the problems I have in understanding the hypothesis that Le Corre and Carey advance. By articulating what I do not understand I may prompt a clarification of these difficult matters. The general thrust of their proposal is to chart a path by which a mind that initially lacks the resources to represent the natural numbers can acquire those resources by building on two other representational systems, which are not specifically dedicated to representing the natural numbers, the enriched system of parallel individuation and the set-based quantification system. My difficulties center on my uncertainty about what the representational resources of these two systems are, particularly the former, which plays the starring role.

The essence of my problem is that I am unclear what representational resources they attribute to their enriched parallel individuation system and why. They say only that it "represents sets of individuals by creating working memory models in which each individual in a set is represented by a unique mental symbol." In their description of the system, it has symbols only for objects, not for sets of objects. That is where my problem begins.

For the parallel individuation system to represent the numerical equality of two sets by determining whether their members may be placed in one-one correspondence, it must have symbols that represent the sets themselves, as distinct from their contents. To recognize and represent sets, it must have the machinery for deciding which objects or symbols are to be included. A set is an entity distinct from its contents. A representation that refers only to the contents of a set does not constitute a representation of the set itself. That is why, for example, in the more or less conventional specification of a canonical sequence of sets by reference to which the cardinality of all other sets may be determined, braces are used to represent the sets themselves, as distinct from their contents. Thus, 'ø' is the symbol for the empty set (or sometimes '{}'), while '{ø}' is the symbol for the set that contains the empty set, and '{ø,{ø}}' is the symbol for the set that contains the first two. The braces are an essential feature of the representational apparatus; they distinguish the sets themselves from their contents. The empty set is not nothing; it is a set that contains nothing. If one fails to distinguish between a set and its contents, the construction of the sequence of canonical sets collapses.

In thinking about this system for attending to a small number of objects, we, who have the symbolic resources to represent sets (mental braces, if you like), may group the symbols in the parallel individuation system into a set or sets, but there is no reason to suppose that the parallel individuation system itself has the symbolic apparatus to do this. It presumably has two different symbol types ('i' and 'i?'), with tokens of the first type referring to established objects of current attention and symbols of the second type referring to newly discovered objects that may or may not be identical with the original objects of attention. By postulation, this system can generate only a few tokens of each type, usually no more than three, sometimes four. Because Le Corre and Carey have distinguished this enriched system of parallel individuation from Kahneman, Treisman, and Gibbs (1992) system of object files and Pylyshyn's (1989) FINST, there is no independent motivation for this assumed limit. It is postulated only to explain the limits they find. As best I can determine, however, the enriched system has no symbol type for sets. Nor can I see any reason why it should have. It is the objects themselves that the machinery of individuation is concerned with, not the various sets into which they might be grouped.

Number is a property of a set of things. It is not a property of the things themselves. Nor is it a property of the symbols that reference those things. Presumably, a symbol system cannot represent a property of something that it cannot refer to. Thus, a representational system lacking symbols that refer to sets is presumably incapable of representing the cardinality of sets. How it is that a symbol system lacking the resources to refer to a set can endow a word with reference to a property (cardinality) that is a property only of sets I do not understand.

I have no difficulty in seeing how the parallel individuation system could direct the search of a box to recover referents for each active token of its 'i' -type symbols. In finding a renewed referent for each symbol of the 'i' type in its working memory, as the child pulls objects out of a box, the system is not conducting a "computation[s] that operate[s] over mental models of small sets." To compute over something requires symbols for that something. As already noted, the parallel individuation system appears to lack the representational resources to refer to sets.

Thus, I do not understand why the parallel individuation system, enriched or not, is said to have "numerical content." A miscellany of objects lying on a table does not have numerical content, nor do the symbols in the parallel individuation system that point to them. Both the objects and the symbols that point to them may of course be treated as the contents of an enumerable set by a representational system capable of representing sets and enumerating them. If being composed of things that could be counted is all that it means to "have numerical content" than literally every conceivable thing and every conceivable collection of conceivable things has numerical content. An internal combustion engine has numerical content.

Similarly, if every process that pairs things one-to-one is a process that establishes and represents numerical equality, then the crystallization of a salt out of a solution is a process that establishes and represents numerical equality, because the negative and positive ions of the salt pair one-to-one as they crystallize out. Numerical equality is a relation that exists only between sets. How can it be established or even represented by a system that lacks symbols for sets?

I also do not understand what they mean by "numerical identity", which the define as "sameness in the sense of *same one*." This seems to me to be a play on the multiple possible meanings of 'one.' In the expression 'same one,' 'one' can be replaced with 'thing' without any loss of meaning, which is not true when 'one' has numerical meaning, as in, "There is one book." Object identity and numerical equality/identity are different things, as may be seen by considering the different constraints on tokens of the 'i' type as opposed to tokens of the numerical types ('1', '2', etc.). An essential constraint on tokens of the 'i' type is that two different tokens never refer to the same object (each object has a symbol unique to it), whereas an essential constraint on two different tokens of a numerical type (say, two different instances of the symbol '1' used to represent a cardinal value) is that they always refer to the same number. Thus, the process of establishing an identity of object reference between, say, 'i_a' and '$i?_b$', is not equivalent to the process of establishing the numerical equivalence of the symbol sets {'i_a'} and {'$i?_b$'}.

Le Corre and Carey may have been misled by the fact that when a symbolic equivalence relation is introduced into first order logic (usually with a view to constructing a representation of the natural numbers), the words 'equality' and 'identity' are often treated as synonyms. The concern in the logic enterprise is to lay down axioms or definitions that establish the condition of complete symbol substitutability. Symbolic equality/identity obtains between two symbols just in case either may be substituted for the other in any valid expression without affecting the value of the function or the validity of the asserted relation. The parallel individuation system, as I understand it, is concerned with object identity, not symbol substitution. Object identity and numerical equivalence are not the same thing: the former implies the latter, but the reverse is not true.

Finally, if the initial meanings that children learn for the words 'one', 'two', 'three' and 'four' are those hypothesized by Le Corre and Carey, I do not understand how these words ever acquire numerical meaning. The proposal seems to depend on a symbolic sleight-of-hand in which object symbols in memory somehow cease to refer to particular objects in order to become an undifferentiated set of tokens, reference to which determines the cardinality of all other sets. The symbols in this special reference set drop their function of picking out particular objects ("each individual is represented by a unique symbol") and take on the function of a set in the canonical sequence of sets used by logicians to establish reference for the natural numbers. When they thus mutate, the motivation for putting distinguishing indices on the tokens disappears. Distinct tokens no longer refer to distinct objects, so distinguishing between tokens by means of indices is no longer appropriate. A set of tokens referring to individual objects has become a set of hash marks, for which the mapping between tokens and objects is irrelevant so long as it remains one-one. This seems to me antithetical to the only thing that has been specified about the symbols in working memory, namely, that each symbol must be unique to a particular object.

Take, for example, their "i_a",[3] which is said to be "the meaning of 'one'." As I read them, a token of the 'i' type in the parallel individuation system refers to a particular object that is currently being attended to. However, at this point in the Le Corre and Carey argument, tokens of this symbol type become symbols in long-term memory. This seems a violation of the just described working-memory function initially imputed to them. Also, now that these tokens are in long-term memory, it is unclear why there is a limit on how many of them may be in some sense active at the same time. (The limit of 3 or 4 was in any case arbitrary, because it does not match the limits of working memory and the enriched parallel individuation system is apparently not Pylyshyn's (1989) FINST system.) That, however, is not my central problem. My central problem is that in migrating to long-term memory, a particular set of tokens, each uniquely referring to a different particular object, somehow becomes a reference model for all sets that have the same number of elements. Consider the token 'i_a', which is said to become the model of a set containing a single file. Initially, 'i_a' is unique to a particular object, A. For the sake of concreteness, let us assume that A was the child's favorite doll. If 'i_a' continued to refer to that particular doll and the word 'one' acquires its initial meaning by reference to the token 'i_a', then the child would presumably learn that 'one' was a proper name for that doll. 'One' would have the same meaning as 'Mary Ann', which the child might otherwise use to refer to her favorite doll.

I am baffled by Le Corre and Carey's assertion that when the child has associated the word 'one' with a particular token 'i_a' of the symbol type 'i', "the meaning for 'one' at this point be the same as the meaning of the singular determiner 'a'." We venture here into deep linguistic waters, where I fear getting out of my depth, but for me the determiner 'a' commonly serves to indicate that the singular noun that it modifies refers to something that has not heretofore been referred to. How a token in working memory whose essential function is to refer uniquely to a particular thing that has already been attended to can give the meaning of the indefinite determiner 'a' to a word associated with it, I do not understand.

In summary, I am disappointed that Le Corre and Carey do not consider the Gallistel and Gelman (1992) hypothesis, for which I confess a partiality. Secondly, I do not understand the hypothesis that they do advance to explain their results, because they say so little about what the representational resources of the parallel individuation system are. Does it have symbols for sets? Are symbols for sets and symbols for objects the only primitive symbols? Or are there others? Does the system have a symbol for numerical equality? How do the symbolic resources available to the system

[3] I delete the braces that Le Corre and Carey place around 'i_a', because their parallel individuation system, as I understand it, has no symbols for sets, only for objects. However, even if we read Le Corre and Carey as intending that '$\{i_a\}$' refer to a symbol in long term memory that refers not to the favorite doll itself but rather to the set of which it is the only member, the problem I am here describing still applies. Now 'one' becomes the name for that set, a set distinguished from all other one-member sets by the fact that its one member is that particular doll.

allow it to define sets (specify the criteria for admission to or exclusion from the sets)? Does the parallel individuation system have any combinatorial mechanisms or principles that enable it to create well-formed formulas (symbol strings belonging to a computable language)? They seem at many points to appeal to notions in first order logic, the implementation of which would seem to require these. For example, the set-based quantification system seems to have the symbolic resources of a quantifier system. These require binding over variables. That would seem to require machinery for generating well-formed formulas. Lastly, if the meaning of a number word is established by reference to a symbol that uniquely refers to a particular object, how can that meaning be a numerical meaning? The essence of numerical meanings is their abstraction from the particular. It is that abstraction that enables us to judge that the number of chairs is sufficient for the number of people present.

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