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Book review

C.R. Gallistel. The Origin of Concepts, S. Carey (Ed.), Oxford University Press, New York (2009). 608 pp., Price: $ 49.95, ISBN: 978-0-19-536763-8

This is a very good book. It covers a huge literature, documenting the rationalist revolution in the conception of cognitive development wrought by basic research on infant and preschool cognitive development over the past four or five decades, the decades spanned by the rise of cognitive science. It is very well written. Students of cognitive science will use it as a reference work, because it contains lucid relatively jargon-free descriptions of so many different lines of research and so many different theoretical positions. Among its many virtues is a late chapter reviewing diverse approaches to a theory of concepts, a topic of very broad interest, for which it is very hard to find any other presentation that does as much justice as Carey's does to the very diverse ways in which philosophers and psychologists conceive of this problem. Its use as a reference work will be greatly facilitated by the fact – increasingly rare! – that it has good and thorough subject and author indices.

The book is not, however, intended as a reference work; rather, it is a sustained argument for a thesis about cognitive development. The dust jacket portrays the thesis in cartoon form. It shows a boy floating above and beyond the edge of a cliff, bent over, pulling on the straps of his boots. We are invited to believe that his pulling on his bootstraps has raised him off the ground on which he presumably once stood. To my mind, the cartoon also captures the essential problem with the thesis, the impossibility of what is proposed.

Carey argues for three theses: The first is that there are two types of knowledge, that is, two types of working representations of the experienced world. The first type is mediated by what Carey (and others) call mechanisms of core cognition. These mediate the representation of the world that seems to be implicit in the behavior of preverbal infants, preschool children, and non-verbal animals. The research that Carey reviews in roughly the first half of the book has shown that these representations appear much earlier and are much richer and more highly structured than had been assumed within empiricist philosophy and psychology. Abstract conceptions of space, time, number, probability, mass, solidity, essence and causality have now been shown to be present in the preverbal child and in non-verbal animals. These findings have moved psychological and cognitive science in a rationalist direction. It is now widely (though not universally) conceded that these abstractions inform behavior-determining representations of experience very early on. They appear to be implicit in some form in our representation of the experienced world from our earliest days.

The second type of knowledge, Carey calls explicit knowledge. She is not as explicit as one might wish about what makes a system of knowledge explicit, but she seems to understand by this a knowledge system that informs what we say when we attempt to talk about how the world works or to represent it in other publicly accessible, socially and historically developed symbol systems like writing and mathematics. (She has a chapter devoted to the pedagogical implications of her bootstrapping hypothesis.) Carey's second thesis is that the development of explicit representations from the implicit representations delivered by the developmentally primitive mechanisms of core cognition is characterized by discontinuities of two kinds. First, the representations in explicit cognition are locally incommensurable with their developmental antecedents. Second, at least in some cases, their development requires the creation of new representational resources, because what is represented in the mature form cannot be represented in the immature form. Her third thesis is that these more mature representation cannot have been learned by any form of hypothesis testing, because what is learned is not expressible using the representational resources available prior to its having been learned. Therefore, she argues, their emergence must depend on a bootstrapping process, a process capable of creating something from nothing.

Carey calls the hypothesized process for creating new and more powerful representational resources Quinian bootstrapping, crediting the philosopher Quine with having first made the argument. She argues that the process has been described in the literature on the history and philosophy of science that is most strongly identified with Kuhn and Feyerabend. This literature maintains that in many instances historically later (better? more mature?) scientific theories cannot be expressed in the language of previous theories, and vice versa. Carey has herself done work in this area. She draws on that work in elaborating her argument.

In particular, she argues that in the early experimental and theoretical work on thermodynamics done by students of Galileo, they did not distinguish between what we now call heat and what we now call temperature. They could not

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conceptualize that a difference in *temperature* causes *heat* to flow from the source with a higher temperature to the source with a lower temperature, because they did not differentiate between temperature and heat. Thus, she argues, our conception, in which these are two different primitives, cannot be expressed in their language. It might be added that a similar state of affairs arose in early experimental and theoretical work on electricity. It took a long time before students of electrical phenomena distinguished clearly between charge, voltage and current.

A clear distinction between what (in treatments at the secondary school level) we now take to be the electrical primitives of charge and voltage was greatly promoted by mathematical formulations (Heilbron, 1979). I suspect the same is true in the history of thermodynamics, but I have never read in the early history of this subject. To my mind, the important role that mathematical formulation played in establishing modern scientific distinctions vitiates Carey's history-of-science examples. It is true that modern formulations of the elementary laws of electricity cannot be formulated in the language that 17th century pioneers used to talk about the electrical phenomena they were investigating, because contemporary language rests on distinctions they did not make when talking about these phenomena. Moreover, had they heard these terms used in our sense, they would not have known how to relate them to what they had observed. They would have faced the problem of establishing their reference, a challenge that still defeats a surprisingly large fraction of contemporary adults, as Carey's review of the pedagogical literature makes clear. Conversely, Ohm's law cannot be stated in a symbol system that does not have distinct symbols for voltage, resistance and current. Thus, for the instruction of modern secondary students in the elements of electricity, one could not use the language that 17th century electricians used to talk about electrical phenomena (at least, not while maintaining what they took to be the referents of terms that may still be in contemporary use). However, there is a more basic language – by which I mean a representationally powerful symbol system in the brain – which was available both then and now, and which may plausibly mediate both historical and ontogenetic/pedagogic change in representation. This is the symbol system by which the brain represents magnitudes of all kinds, including the abstract magnitudes of number, distance, duration, rate, proportion and probability.

There is now extensive empirical reason to believe that the ability to represent magnitudes and manipulate them arithmetically is primitive, both evolutionarily and ontogenetically. Indeed, Carey discusses this *analog magnitude* system first among the three core representational systems from which she argues that the adult conception of number is bootstrapped. She reviews some of the evidence for its existence in both preverbal children and non-verbal animals and for its role in the adult cognition of number. However, she then argues that it plays next to no role in the development of the adult conception of number. From there on out, she ignores the role that it plausibly plays in the cognitive development she is concerned to understand. This is odd, because she repeatedly alludes to the importance of analogy in bootstrapping, particularly in her discussion of the historical examples. In doing so, however, she ignores the fundamental role that simple arithmetic relations (for example, order, difference and proportion) play in these analogies – both explicitly and, more importantly, I would argue, implicitly. When one says – as generations of teachers have – that the elementary electrical and thermodynamic quantities of charge and heat map to the hydrodynamic quantity of amount of fluid and that voltage and temperature map to pressure (or "drop", that is, from a high to a low level of water), and that this mapping preserves the formal (arithmetic) relations between these quantities, many students who are completely buffaloed by the algebra – simple as it is – nonetheless acquire a useful intuitive understanding of electricity. Underlying the intuitive understanding thus gained is very probably the system for representing magnitudes, which system they already use to effectively represent more directly perceptible aspects of the experienced world. You cannot see the heat flowing or the charge flowing, but you can see the water flowing from the higher to the lower level and perceive that the amount of water at the higher level is not the same as the drop between the two levels. Clearly one magnitude cannot be used to represent both of these perceptually different aspects of the experienced world.

Carey seems to forget about the representational power inherent in the analog magnitude system when she later writes that "Before the representation of the rationals [which, she argues, comes only well into the school years and then only to some] children cannot think thoughts about 3/4, .75, 79/80 or .9875."—(p. 353) or when she writes "there are no symbols in core cognition with the content *1* or *0* or *add* (although this content is embodied in the computations defined over parallel individuation and analog magnitude representations together)."—p. 528.

Rats represent rates (numbers of events divided by the durations of the intervals over which they have been experienced) and combine them multiplicatively with reward magnitudes (Leon and Gallistel, 1998). Both mice and adult human subjects represent the uncertainty in their estimates of elapsing durations (a probability distribution defined over a continuous variable) and discrete probability (the proportion between the number of trials of one kind and the number of trials of a different kind) and combine these two representations multiplicatively to estimate an optimal target time (Balci et al., 2009). Human adult subjects generalize from the proportion between two durations to the proportion between two integers without being instructed to do so (Balci and Gallistel, 2006). These are but a few of the many experimental results that imply that the analog magnitude system represents both discrete and continuous quantity and brings to bear on the symbols that refer to these abstract aspects of our experience the representational power of the arithmetic field (the system of arithmetic that is closed under addition, multiplication and their inverses).

How can there not be symbols for *1* and for *0* in the analog magnitude system? They are the multiplicative and additive identity elements, respectively. The symbol for the multiplicative identity has the unique property that when fed to the multiplication machinery along with any other symbol in the system (including another instance of the identity symbol), what comes out is that other symbol. Moreover, this symbol must map to (refer to) the numerosity of a set with one element, or the mental books will not balance (Gallistel, in press). Similar considerations apply to *0*. It is the symbol for the result of subtracting one instance of a mental magnitude from another instance of that same magnitude. It is also the magnitude, at which sign (the direction of a magnitude) reverses.

When subjects mentally subtract the mental magnitude for one estimate of numerosity from the mental magnitude for another, the resulting magnitude has sign. That is, it is a directed magnitude; the difference may favor the subtrahend or the minuend, depending on which is larger. Or, it may favor neither, if the two estimates are the same. Subjects do this task readily (Cordes et al., 2007). Importantly, there is no evidence of a discontinuity in their performance at the additive identity, the difference at which the sign (direction) reverses. This would seem to imply that there is no special machinery for dealing with the case where the difference is 0. There would have to be special machinery for this if a symbol for 0 could not be generated by the analog magnitude system, because in that event, the system would not be closed under subtraction. There would be inputs to the symbol-crunching machinery for which the machinery was incapable of generating an output, inputs that "crashed" the machine. Historically, in verbally mediated arithmetic reasoning, exactly this state of affairs obtained. Absent written symbols for 0 and negative numbers, the historical system of verbally mediated arithmetic was not closed under subtraction. This lack of closure made it a very awkward system to work with. But, to repeat, the evidence from experiments in which subjects are asked to perform subtraction with the analog magnitudes representing the numerosity of sets gives no reason to believe that this core system is similarly encumbered.

Similar arguments apply to symbols for the rational numbers. A brain that represents rates (that is, proportions between numerosities and durations) and probabilities (proportions between two numerosities) with analog magnitudes could barely function if it could not generate and operate on magnitudes to represent non-integer quantities. This representational system would not be closed under division. Lack of closure under division would mean that it could not represent proportions, rates and probabilities. Historically, this was the case for the verbally mediated system of arithmetic reasoning. The fact that the early overtly symbolized system of arithmetic reasoning was not closed under division greatly limited its representational potential, as indicated by the reasoning recorded in ancient Egyptian mathematical texts. Even then, those engaged in such reasoning seemed to intuit that there ought to be arithmetically manipulable symbols for the proportions between integers. Their problem was that they did see how to devise a notation capable of explicitly generating the requisite written symbols and making them reliably subject to arithmetic manipulation. A system for doing this was not devised for thousands of years thereafter. Moreover, as Carey notes, a discouragingly large fraction of students in the United States finish high school (and even college) without ever having mastered this system.

The inability of many students to learn to deal with fractions should not be taken as evidence that their brains are incapable of representing, for example, probabilities. Carey takes it as self-evident that the brain of a preschool child cannot think thoughts about .75, but this is conventional notation for a magnitude that may represent (among many other possibilities) 3 chances out of 4. We know that the mouse can think quite sophisticated thoughts about how this probability should be combined with the mouse's representation of its uncertainty regarding the duration of an elapsed interval in order to choose the optimal decision point in a timing task (Balci et al., 2009). It seems implausible that the human child cannot think thoughts about *what may be denoted by* .75, however, incapable it may be of understanding the complex way in which this overt notation represents that simple magnitude.

Carey repeatedly ignores the role that the core system of reasoning with analog magnitudes (aka real numbers) demonstrably played in the history of scientific thought and may plausibly play in cognitive development because she believes that the system of analog magnitudes plays very little role in the development of adult numerical cognition. Her treatment of this development is the center piece of her argument that development creates, via bootstrapping, representational systems that have more representational power than the innate core systems of representation with which cognitive development begins. Before describing that treatment and what motivates it, it will help to briefly sketch an alternative story that could be told.

The alternative story assumes that the human genome contains genes that together specify the structure for an evolutionarily ancient mechanism that implements arithmetic. That is, it implements >=, +, −, *, and ÷) on symbols that may refer to experienced magnitudes (distances, speeds, durations, numerosities, weights, volumes, areas, etc.). This system enables an animal's brain to represent the animal's location relative to the location of other behaviorally important locations in its experienced world, to represent the durations of behaviorally important intervals, to represent the cardinal values of behaviorally important sets, to represent behaviorally important probabilities and uncertainties, and so on.

Wigner (1960) called attention to "The unreasonable efficacy of mathematics in the natural sciences." Mathematics may be seen as the continuing exploration of the representational potential of arithmetic. The spooky ability of symbolic systems built on this simple foundation to represent the experienced world is a reason for supposing that there might be an evolutionarily ancient implementation of arithmetic in the machinery of the brain. This rationale is, of course, quite aside from the extensive behavioral evidence for a system of arithmetically manipulable magnitudes—some of which Carey reviews. Moreover, there is a quite general consensus, to which Carey subscribes, that such a system plays an important role in mediating practical numerical reasoning in adults (Dehaene, 1997).

On the alternative developmental story, the development of adult numerical cognition consists in learning how conventional systems for the overt public symbolization of quantity (number words, numerals, fractions, decimals, algebraic expressions, etc.) map to this powerful system that the brain already uses to represent such basic aspects of everyday experience as where home is and how long it takes to get there from where one is. Because the innate system already has all the representational power of the mature overt, culturally developed system, there is no increase in the representational potential of the underlying core system, the system that mediates unverbalized thought about quantity. Rather, the overt system acquires the representational potential already present in the innate system. This acquisition may then mediate the extension of that potential into realms where the innate system would not otherwise operate effectively. Although, there are reasons to suppose that a system of arithmetic reasoning may be innate, no one supposes that the mathematical representation of relativistic and quantum mechanical phenomena are. These representations exploit the representational potential inherent in arithmetic rather than enabling us to represent things that are not representable in arithmetic.

Carey argues that adult numerical cognition develops from three core systems: (1) the analog magnitude system; (2) the system of parallel individuation; and (3) set-based quantification. Her treatment is pretty clearly motivated both by the logicist tradition in the area of mathematical foundations (the failed attempt, pioneered by Frege, to anchor arithmetic, hence all of mathematics, in logic and set theory) and in the history of mathematics. She argues that, although most children by the age of three or a little later can reliably count to 10 and often much farther, they do not understand what they are doing: they have no notion that this procedure generates a representation of the cardinal value of a set. Her evidence for this comes primarily from a task in which children are asked to give the experimenter *n* items. When $n = 1$ (“Give me one toy”), most children reliably give the right number by the age of two and a half. However, only at much later ages do they reliably give the right number when $n = 4$. Carey calls children who reliably give the right number when asked to give one item “one-knowers”, those who also do so for two are called “two-knowers”, and so on. She insists that a “two-knower” that can also reliably count correctly to two and even well beyond nonetheless does not understand that counting yields a representation of cardinality. In insisting on this, she ignores considerable evidence to the contrary (for review, see Gelman and Gallistel, 1978). The simplest and perhaps most powerful evidence to the contrary is that when a judgment of the cardinality of a small set (2 or 3) is called into question, children aged two and one half and three count the set whose cardinality has been questioned (Bullock and Gelman, 1977; Gelman and Meck, 1992; Gelman, 1993).

Carey argues that the meaning of ‘two’ for a “two-knower” – the nonlinguistic symbol with which the word becomes associated, thereby enabling the child to use the word correctly (in response, for example, to the request, “Give me two.”) – is a set of (what were once) symbols in the parallel individuation system. Carey early on reviews the literature showing that the brain has a system for tracking objects, with a capacity that is limited to approximately four objects (Kahneman et al., 1992; Pylyshyn, 2001; Vogel et al., 2001). This system has symbols for the objects that are being tracked. These symbols point at particular objects. They are inherently evanescent, because the objects being tracked change on a short time scale. Carey believes that sets of these evanescent, pointer symbols in working memory can somehow become stored in long term memory, shed their original function, which was to point at a particular object, and then serve the function that Frege’s canonical sets served, that is, to establish numerical reference. She argues that in the mind of the three-year old, a set has numerosity two just in case its members can be placed in one-one correspondence with a set of sometime pointers that happens itself to have two members.

This is where Carey’s argument makes contact with Frege’s set-theoretic construction of the positive integers. Frege defined two as the set of all sets that can be placed in one-one correspondence with a canonical set with two members, namely, the empty set and the set that contains only the empty set. Carey follows Frege in treating the establishment of reference as the first step in a theory of concepts. This focus on the problem of reference is consistent with the historical record. In the days when many mathematicians thought there was no such thing as a negative number, they thought this because they did not believe that a referent could be assigned to a negative number.

Carey is clear that in the parallel individuation system “. . .there are no symbols for number at all. . .”—p. 138. And “These symbols represent the individuals in the set, not the number of them (except implicitly, for there is one symbol for each individual).”—p. 141. As is evident in the second of these two quotes, she argues nonetheless that this system has “implicit” numerical content. Her argument rests on experimental findings showing that infants keep track of the location of several different objects, even when the objects are occluded. This seems to me entirely irrelevant. The function of these pointer symbols is to pick out particular individuals. What these experiments show is object permanence: the brain of the infant represents objects as continuing to exist even when they can no longer be perceived, and it does so even when there is more than one object. On almost any story about object permanence, there would have to be symbols that pointed at or referred to objects. The more objects being tracked, the more such symbols must come into play. But the currently active pointing symbols no more represent the numerosity of the set of objects they point to than the pistons in an internal combustion engine represent the numerosity of the set of cylinders they move in. Under no story about representation are the pistons a representation of the numerosity of the cylinders, implicit or explicit, even though the pistons can always be placed in one-one correspondence with the cylinders. A symbol has a numerical content *iff* it enters into (at least some) of the above listed operations that define the system of arithmetic. Placement into one-one correspondence is not on the list; it is at most an operation for establishing reference.

The third core system from which the adult conception of number emerges is, Carey argues, set-based quantification. This system is closely tied to language; it is the system that, Carey argues, gives meaning to the quantifiers in language. I have trouble following her reasoning here. I am not clear what the properties of this system are, what its symbols may refer to, and what operations they may enter into. Carey argues that the role for this system in the bootstrapping induction that yields the concept of the natural numbers is to supply explicit symbols for *set*, *individual*, *singular*, *plural*, *dual*, and *triple*, which concepts, she argues, “are only implicit or absent in parallel individuation and analog magnitude representations.”—p. 329

The complex account of how the concept of the natural numbers arises from these three core knowledge systems by bootstrapping is at the core of Carey’s developmental thesis: “The integer list is a cultural construction with more representational power than any of the core representational systems on which it is built, thereby providing a genuine

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counterexample to the argument that conceptual discontinuities are in principle impossible. Chapter 9 sketches out the later (in history and ontogenesis) development of the concept *rational number*, which equally well transcends the representations available at the outset of the construction process (namely, representations of integers created by children during their preschool years). In these cases, discontinuity is cashed out in terms of vast increases in expressive power"—p. 19 The thesis is that this bootstrapping process in which language plays a critical role is a "uniquely human learning mechanism that underlies the human capacity to create new representational resources – that is, to create concepts not available in *or definable in terms of* antecedent representations." – p. 246 [italics mine]

The claim that the new representational resources are not definable in the antecedent representations is the core claim about bootstrapping. It sets bootstrapping apart from all forms of hypothesis testing, including, for example, Bayesian model comparison. Hypothesis-testing models of concept development require an antecedent symbolic language in which the competing models are expressed. Hence, the representations never transcend the representational power of this antecedent symbolic system.

The first problem with the bootstrapping account of how the concept of the natural numbers emerges from the three above-described core systems is that it begs the inductive question, as has been explained at length by Rips and colleagues (Rips et al., 2006, 2008). They show that the posited induction is not constrained to end up with the result it does end up with, either by the data on which it is presumed to operate nor by any constraints in the inductive mechanism. Carey does not specify the inductive mechanism. Rips et al. argue that the only way to get the desired result is to build it into the inductive mechanism, in which case the resulting system does not transcend the antecedent system.

There is an even deeper problem with the further account of how the concept of a real number develops from the concept of a natural number. The account is intended to illustrate the claim that bootstrapping can create a representation that was not definable in the antecedent symbol system. The problem is that real numbers *are* definable in terms of the natural numbers. Showing how to do this was what the development of number theory in the 19th century was all about. Moreover, the reverse is not true: despite the fact that the natural numbers are among the real numbers, the natural number system is not definable within the real number system. In Carey's terms, this means that the system of the natural numbers has more expressive power than does the system of real numbers. In the system of real numbers, one cannot express the notion of *next*, whereas in the system of natural numbers one can. This is, in essence, why the system of real numbers is Gödel complete, whereas the system of natural numbers is not. Incomplete symbolic systems have more representational power than complete ones. The additional representational power is what gets them into trouble, so to speak, making them incomplete. (A symbolic system is incomplete if it must contain well formed expressions such that neither the expression nor its negation can be derived within the system.)

For the reasons sketched above, I am deeply skeptical about Carey's central thesis. This in no way alters my admiration for her book. It is the best thing on concepts that has appeared in decades. It is clear, thought provoking, and broadly informative about experimental results.

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