# Basic Set Theory

DEF: For our purposes, a **set** is any collection of entities. Sometimes we list the elements in a set:

{John, Bill, Mary}
{5, 77, 12, 1818}
{1, 3, 5, 7, 9, . . . }

Sometimes we describe the set by using some kind of property:

{*x*: *x* is an even number} = the set of even numbers

{*x*: *x* is a number} = the set of all the numbers (and only the numbers).

DEF: x $\in$ A iff x is one of the elements in A.

Fred $\in$ {Fred, Tom}
7 $\in$ {*x*: *x* is a number}
7 $\in$ {*x*: *x* is an even number}

The empty set (written ¯ or {} or $\varnothing$) can be defined: $(\forall x)x \in$ ¯

DEF: For any sets A and B, A is a **subset** of B (written A $\subset$ B) iff everything that is in A is also in B: $(\forall x)(x\ \ A \supset x \in B)$

Thus, for any set A, A $\subset$ A

{7, 6, 8} $\subset$ {3, 4, 8, 55, 7, 6, 4}

{Ronald Regan, Jimmy Carter} $\subset$ { *x*: *x* is a former President}

{*x*: *x* is prime} $\subset$ {*x*: *x* is a number}

Sets are identified solely by their elements:
For any sets A and B:

A = B iff A $\subset$ B and B $\subset$ A.
That is, A = B iff A and B have exactly the same elements.

{1, 3, 5, 7} = {*y*: *y* is an odd number and *y* < 8}

{3, 4, 5, 6} = {6, 3, 5, 4} =

{6, 6, 6, 5, 3, 4, 5}

*Question:* Let A be any set. $\emptyset \subset A$?
*Answer:* Yes. why?

DEF: **N** is the set of natural numbers:
**N** = {1, 2, 3, . . . }

DEF: A set A is **finite** iff there is some natural number *x* such that the elements of A can be paired up with the first *x* natural numbers. Otherwise A is **infinite**. (We also say that the empty set $\emptyset$ is finite.)

The following sets are finite:

{Bill, Hilary, George, Al}
{x: x is a town in New Jersey}
{x: x is even and x is prime}

**N** is infinite. (Obviously?!)

Notice, too, that the set E of even numbers is infinite, too.

*Proof.* Suppose E is not infinite. Then E is finite, so there is some natural number *x* such that the elements of E can be put on the sequence:
1, 2, 3, . . . *x*. But obviously, all the members of the sequence 2, 4, 6, . . . 2*x* are members of E, and so is *2x+2*, which is a sequence of *x+1* numbers. So they cannot be paired up with the first *x* numbers. *x* was an arbitrary natural number, so the case holds for any natural number. So there is no number *x* such that there are exactly *x* elements in E. Contradiction. So E is not finite. So E is infinite.

*Kent Johnson, 2000*