Truth Tables and Mathematical Induction

Question: Why can we be sure that this procedure exhausts the possible combinations of truth values (for any collection of simple statements $\alpha_1,...,\alpha_n$)?

For any such collection of statements, what we want to do is to construct all the possible n-ary sequences of Ts and Fs.

We will answer our question by means of a technique known as mathematical induction. (See notes on mathematical induction.)

Base Step. Suppose n=1. Then there is only one statement letter α_1 in Γ.
By our logical assumption, α_1 is T or F ($2^1 = 1$)

Induction Step. Suppose we have constructed all the possible sequences of length m. We now want to construct all the possible sequences of length m+1.

To do this, it is enough to construct all the possible m-ary sequences and then put a T at the end of each one of them, and then to construct all the possible m-ary sequences again, this time putting an F at the end of each one of them.

Thus, the number of (m+1)-ary sequences is *twice* the number of m-ary sequences. Since there are 2^m m-ary sequences, there are $2^m * 2 = 2^{m+1}$ (m+1)-ary sequences.

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