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Bias Toward Regular Form in Mental Shape Spaces

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The distribution of figural "goodness" in 2 mental shape spaces, the space of triangles and the space of quadrilaterals, was examined. In Experiment 1, participants were asked to rate the typicality of visually presented triangles and quadrilaterals (perceptual task). In Experiment 2, participants were asked to draw triangles and quadrilaterals by hand (production task). The rated typicality of a particular shape and the probability that that shape was generated by participants were each plotted as a function of shape parameters, yielding estimates of the subjective distribution of shape goodness in shape space. Compared with neutral distributions of random shapes in the same shape spaces, these distributions showed a marked bias toward regular forms (equilateral triangles and squares). Such psychologically modal shapes apparently represent ideal forms that maximize the perceptual preference for regularity and symmetry.

Shape classification, like many classification tasks, can be regarded as a decision among somewhat fuzzy categories. In contemporary views of category structure stemming from Posner (Posner, Goldsmith, & Welton, 1967; Posner & Keele, 1968) and Rosch (1975; Rosch & Mervis, 1975), categories are regarded as graded structures built around a central prototype, with less typical members inhabiting the category's periphery, or around some set of highly typical exemplars (Medin & Schaffer, 1978; Nosofsky, 1988). It is natural to imagine that shape categories would also be structured this way. Shapes can exhibit enormous variety within a single shape category, a fact that creates tremendous problems for shape classification systems. One might well imagine that among the diverse members of a given shape category, there might be more and less typical examples. Indeed, studies of learning in dot pattern have shown that pattern classes tend to be organized around some set of prototypical configurations (Peterson, Meagher, Chait, & Gillie, 1973; Posner et al., 1967; Shin & Nosofsky, 1992). Yet, in the specific case of shape, remarkably little is known about the typicality structure of categories.

Mental Shape Distributions

Formally, a shape category may be regarded as a *distribution* (e.g., of probability or of typicality) over shape parameters, perhaps exhibiting a central mode at some maximally prototypical object and tailing off at more atypical objects[1] (Ashby & Gott, 1988; Fried & Holyoak, 1984; cf. Shepard, 1987). This distribution would play a crucial role in any context in which an observer must decide whether an observed shape belongs to a given class under conditions of uncertainty. For example, in recognizing a shape from a noisy image (Lowe, 1987), finding the "object" in a cluttered image (Jacobs, 1996), or perceptually completing an occluded shape (Buffart, Leeuwenberg, & Restle, 1981; Sekuler, Palmer, & Flynn, 1994), the observer needs a measure of how well the observed image data fits within each of the various candidate shape classes. In such a situation, the category distribution fills this role by measuring how likely an outcome the observation would be as a result of each category hypothesis.

More specifically, consider an observer who must decide among k shape classes $C_1 \ldots C_k$ (e.g., triangles, telephones, kangaroos, etc.) given an observed configuration x. From a Bayesian point of view (see Duda & Hart, 1973, or Knill & Richards, 1996, for perceptual applications), the probability that the configuration actually belongs to the class C_i is the *posterior probability:*

$$p(C_i|x) = \frac{p(x|C_i)p(C_i)}{p(x)},$$

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[1] This description is intended to capture one aspect of the mental representation of the shape category. Clearly, it does not capture all aspects of observers' knowledge of shape: For example, it does not capture observers' knowledge that a three-sided polygon, no matter how atypically shaped, is still a triangle. The dichotomy between observers' knowledge of clear-cut rules determining category membership and their impression that more prototypical examples are "better" category members is discussed at length by Armstrong, Gleitman, and Gleitman (1983).

which expands to

$$p(C_i|x) = \frac{p(x|C_i)p(C_i)}{p(x|C_1)p(C_1) + p(x|C_2)p(C_2) + \cdots + p(x|C_k)p(C_k)},$$

where $p(C_i)$ is the prior probability that the shape belongs to the i-th class. These priors are usually unknown and are often assumed to be equal for all C_i (the so-called "principle of indifference"). The crucial terms in the equation are thus those of the form $p(x|C_i)$, usually referred to as "likelihood" terms. The likelihood term $p(x|C_i)$ denotes the probability that the configuration will take the form x given that it really belongs to the i-th class. That is, for a fixed shape x, the likelihood of x measures how well x fits in the i-th class. Allowing x to range all over the shape space, the likelihood specifies the complete distribution of shape within the i-th class. Hence, it is natural to take the likelihood term as denoting the typicality gradation function within the given shape class under the assumption that more typical shapes are more probable. This distribution is probably best regarded as a *subjective* probability distribution, representing the observer's beliefs rather than objective facts; its somewhat subtle relation to probabilities in the world are discussed later. This distribution can be regarded as giving probabilistic meaning to the gestalt notion of "goodness of form," in effect specifying the complete distribution of figural goodness (cf. Van der Helm & Leeuwenberg, 1996).

This article reports experiments designed to map out the shape likelihood functions within two simple shape spaces: the space of triangles and the space of quadrilaterals. Triangles and quadrilaterals are an admittedly specialized subset of natural shapes, but their simplicity and low dimensionality make possible a careful study that would be impossible with arbitrary shapes. Two types of methodology are used: In the *perceptual methodology,* observers are asked to rate the typicality of shapes presented on the computer screen. In the *productive methodology,* observers are asked to draw examples of shapes from a given category. These two tasks are taken as independent routes to estimating the structure of the shape typicality distribution.

It may seem peculiar to expect a typicality gradient on triangle and quadrilateral spaces because, after all, from a geometric point of view any given object either is or is not a member of each category. However, this fact need not preclude substantial variations in subjectively judged typicality (Armstrong, Gleitman, & Gleitman, 1983).

Mathematically, triangles and quadrilaterals can each be regarded as well-defined parametric spaces of dimensions 2 and 4, respectively (see Robertson, 1977; because I am interested primarily in shape per se, I ignore position, orientation, and scale). This means that any triangle can be specified by exactly two numbers—each triangle has two "degrees of freedom"—and any quadrilateral by exactly four.[2] Each space can be parameterized in many different ways (cf. Krantz & Tversky, 1975; Shepard & Cermak, 1973), but different choices of parameterization simply mean different ways of indexing the same basic two- or four-dimensional space. For convenience, I adopt a consistent parameterization for each space throughout this article, as follows.

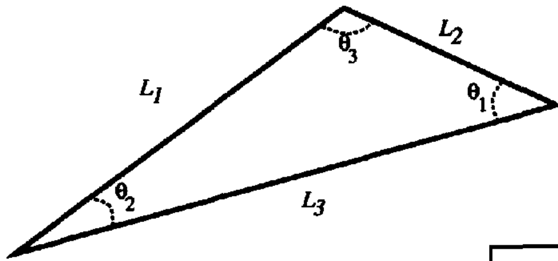
Any triangle can be parameterized by an interior angle θ and the ratio R of the two adjacent sides (taken as the quotient of smaller over larger and hence lying between 0 and 1). Each triangle has three such parameterizations corresponding to the three angles. Because there is nothing to choose among them, in the analysis below I use all three of these simultaneously (so that each triangle shows up three times in the tabulations). For quadrilaterals, the parameters are the aspect ratio S (length of shorter axis divided by length of longer axis), the skew α (positive angular deviation from rectangularity averaged across all directions), and the two tapers τ_1 and τ_2 (taper is the deviation from parallelness of opposite sides and measures 0° when they are parallel). Figure 1 shows geometric interpretations of all the parameters, and Figure 2 shows schematic illustrations of the two spaces. Again, it should be emphasized that θ and R completely specify an arbitrary triangle, and thus the space (θ, R) can be thought of as a "natural home" for triangles and likewise $(S, \alpha, \tau_1, \tau_2)$ for quadrilaterals.

It is worth remarking that each of these spaces is almost exclusively populated by "scalene" or irregular examples. Isosceles, equilateral, and right triangles, as well as squares, rectangles, parallelograms, and so forth, are all of measure zero in the respective overall spaces, meaning that a randomly selected example has probability arbitrarily close to zero of having any of these special properties. The mathematically modal shape is completely "irregular." However, whether *psychologically* modal triangles and rectangles are *mathematically* modal is, of course, an empirical question.

In looking for modes and other psychological structure in these spaces, though, one must be careful about one important technical point. These spaces are not inherently uniform from the point of view of probability density. In other words, if triangles and quadrilaterals are selected at random, the resulting density in the limit will not be uniform in (θ, R) or $(S, \alpha, \tau_1, \tau_2)$. This problem is discussed at length in the pioneering statistical work of Kendall and Kendall (1980). Even the notion of choosing shapes at random is notoriously slippery, a point illustrated by the Bertrand paradox (see Duda & Hart, 1973). In this problem, a seemingly simple question of geometric probability is given three apparently convincing but inconsistent answers stemming from three different but all apparently reasonable methods of defining "random" selection of geometric objects. In the resolution of the paradox (see Jaynes, 1973),

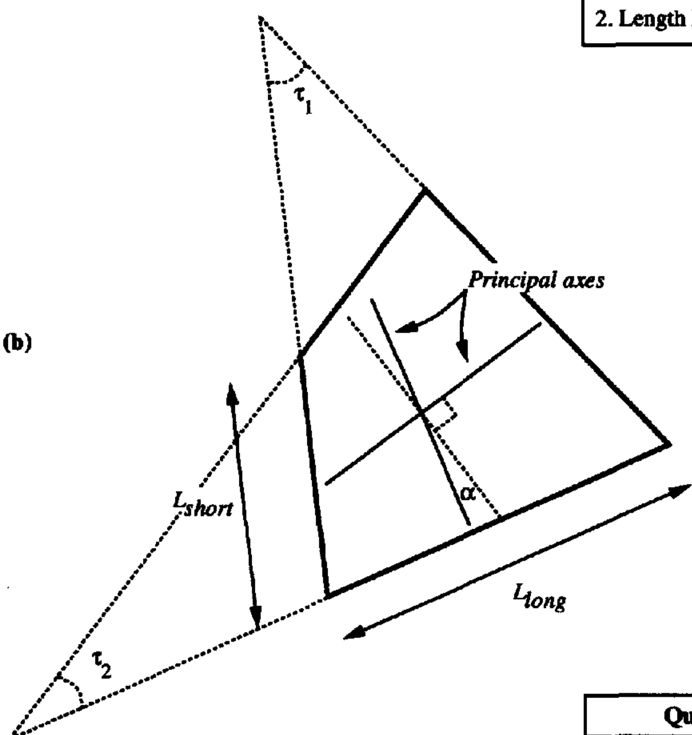
[2] A simple way to see this is as follows: A triangle consists of three planar points, each specified by two numbers, x and y, for a total of six degrees of freedom. From these we subtract two degrees of freedom for translation in the plane, one for rotation and one for scale, leaving two. Likewise, for quadrilaterals, $(4 \times 2) - 2 - 1 - 1 = 4$.

(a)



Triangle Parameters	
Name	*Symbol*
1. Angle	θ
2. Length Ratio	$R = L_i/L_j$ for $L_i < L_j$

(b)



Quadrilateral Parameters	
Name	*Symbol*
1. Aspect ratio	$S = L_{short} / L_{long}$
2. Skew	α
3. Taper 1	τ_1
4. Taper 2	τ_2

Figure 1. Parameters used for (a) triangles and (b) quadrilaterals.

it is seen that two of the methods are actually subtly defective in that they are not invariant to transformations that ought not to change the answer (e.g., translation and rotation of the entire problem).

Hence, to inspect psychological distributions of shape in a legitimate way, one needs to create a truly mathematically neutral shape distribution and then compare the empirical distribution to it. The difference between the human distribution and the neutral one can be regarded as the true mental bias (cf. Foster, 1983). Only by examining this differential bias can one properly gauge the structure of the psychological category.

Following the discussion of Kendall and Kendall (1980) and Jaynes (1973), I created mathematically neutral shape distributions using the following method: First, random points were generated in the plane using a circular Gaussian distribution. This choice is mandatory to establish both rotational invariance and independence of x- and y-

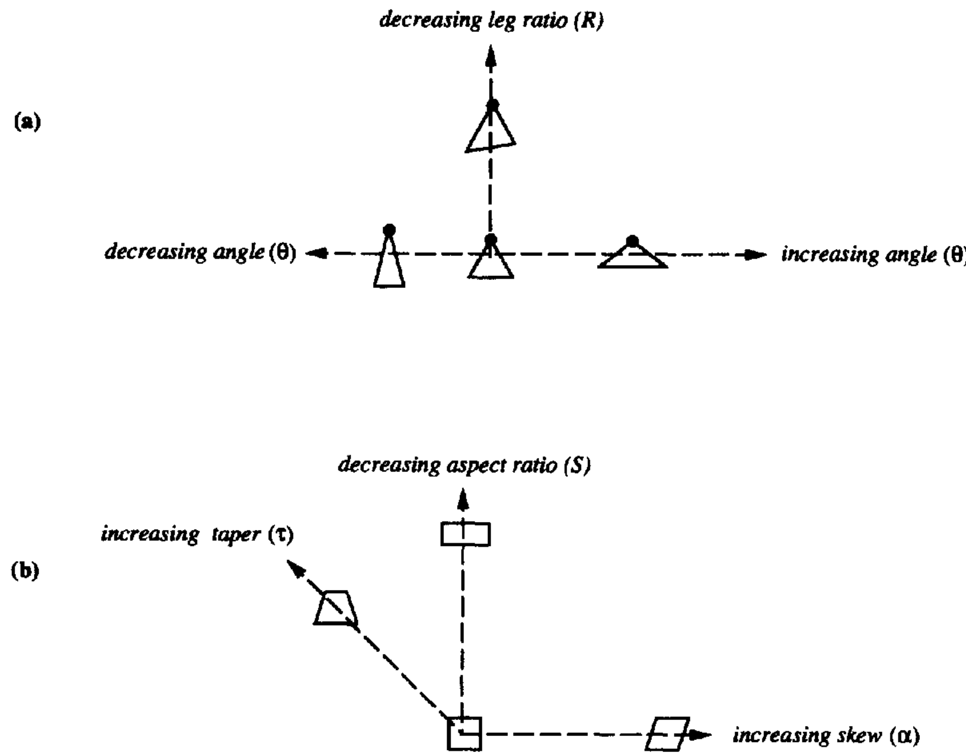


Figure 2. Schematic illustrations of triangle space (θ, R; Panel a) and quadrilateral space (S, α, τ_1, τ_2; Panel b). Quadrilateral space is four-dimensional and cannot be completely depicted in two dimensions. Here a single taper τ stands in for the two tapers τ_1 and τ_2. The other τ is identical, except taken in the other direction.

coordinates[3] (perhaps surprisingly, the bivariate Gaussian is the unique circularly symmetric probability distribution in which x and y are independent). Triplets (or in the quadrilateral case, quadruplets) of points were generated to create triangle (quadrilateral) distributions. Quadrilaterals were required to be convex[4] so that exactly the same distribution of shapes could be presented to observers in the perceptual task (I felt that concave quadrilaterals would be regarded by observers as an entirely separate type). Finally, the two (respectively, four) parameters were extracted and frequencies tabulated to create the final neutral distribution of shape with respect to each parameter. These distributions are plotted alongside the data in the figures.

In Experiment 1 I used the perceptual task, in which observers were asked to rate the typicality of shapes generated by the procedure described above. In Experiment 2 I used the production task, in which observers were asked to draw examples of the two shape categories after viewing a single example. The results from the two experiments are presented and discussed together below.

Experiment 1

Method

Observers. There were 12 observers in the triangle condition and 12 observers in the quadrilateral condition. Observers were members of the university community paid for their participation or students receiving credit in an introductory psychology class.

Procedure. Each observer was presented with 800 shapes generated by the random procedure described above. Each observer saw a different random sequence of shapes. Observers were instructed to rate the "typicality" of the shapes, regarded as members of their respective categories, on a scale of 1 to 7 (1 = *not at all typical*, 7 = *very typical*).

Shapes were black outlines drawn on a white screen at high contrast, subtending about 2.5° of visual angle. Observers were free to move their eyes and to take as long as they wanted to respond.

Results

The results of Experiment 1 are discussed along with those of Experiment 2.

Experiment 2

Method

Observers. There were 148 observers in the triangle condition and 115 observers in the quadrilateral condition.

[3] However, it turns out that in general random shape distributions are not very sensitive to changes in the way the points are distributed (Kendall, 1985).

[4] Quadruplets of points were generated by the procedure described above and then discarded if they passed a test for concavity.

Procedure. Observers were shown a single generic triangle or quadrilateral as an example (see Figure 3) and were asked to draw a number of different examples of the category (with triangles, 6; with quadrilaterals, 10) in boxes provided in a paper-and-pencil booklet. The data were collected after an unrelated perception experiment or in psychology classes.

Analysis. A total of 877 codable triangles and 1,106 codable quadrilaterals were collected in Experiment 2. In addition, 1.2% of the triangles and 3.8% of the quadrilaterals were uncodable and were discarded. Shapes were considered uncodable if their sides did not meet to within measurement tolerances or, more rarely, if they had the wrong number of sides. Vertices were measured by hand and the shape parameters extracted by computer based on the positions of the vertices.

Results

Figures 4–6 show the perceptual subjective typicality distribution (Experiment 1), the productive subjective typicality distribution (Experiment 2), as well as the mathematically neutral distribution, all plotted together for each of the two triangle parameters (see Figure 4) and four quadrilateral parameters (see Figures 5 and 6). Note that in each plot the left y-axis indicates judged typicality (Experiment 1), whereas the right y-axis indicates frequency (Experiment 2). (The production data have been scaled to have the same maximum as the perceptual data; the neutral density curve has been scaled to have the same area under the curve as the production data histogram, meaning that they can both be taken as probability density functions.) The difference between the empirical distributions and the neutral distributions, which in some cases is apparent, can be taken to represent the psychological bias in these spaces.

In most cases, the psychological distribution curves are modal, with a clear prototypical case and degradation at more eccentric cases. Moreover, in most cases, the perceptual and productive curves show the same mode, although the perceptual curve almost always shows a broader peak and a more gradual descent. This is to be expected considering that observers in this task were shown the full gamut of random shapes, including shapes that one might imagine would never come to the minds of the observers in the production task.

Triangles. In the triangle case, the psychological bias along the angle variable θ was apparent (see Figure 4a). The mode in both perceptual and productive cases was at about 55°–60°, whereas the mathematically neutral mode is at 5°. The sample triangle (angles of 20°, 40°, and 120°) seemed to have exerted no influence at all.[5] For length ratio R, both the psychological modes fell at or slightly below unity, whereas the less peaked neutral distribution reaches a maximum at about .70.

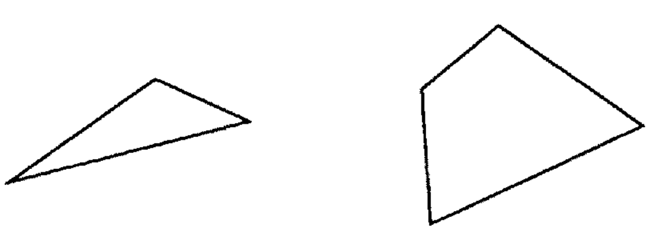


Figure 3. Sample triangle and quadrilateral shown to observers in Experiment 2.

Hence, remarkably, although the mathematically modal triangle was scalene, the psychologically modal triangle was nearly equilateral ($\theta = 60°$, $R = 1$; see Figure 7). This is discussed below.

Quadrilaterals. Here the skew α showed the clearest bias: The neutral mode was at 45°, whereas the perceptual mode was at 0° (no skew; i.e., rectangular) and the productive mode was at about 25°. For aspect ratio S, the productive and neutral modes were in close agreement at a bit less than unity, whereas the perceptual distribution showed no clear mode (the only curve in which this was so). For the two tapers τ_1 and τ_2, all curves showed fairly clear modes at about 0°. Again, overall, the sample shapes seemed not to have affected the locations of modes substantially.

In short, the psychologically modal quadrilateral was apparently a square ($S = 1, \alpha = \tau_1 = \tau_2 = 0°$; see Figure 7), with perhaps a secondary mode at the 25° rhombus ($S = 1$, $\alpha = 25°$, $\tau_1 = \tau_2 = 0°$; see Figure 7). The relatively flat aspect ratio curve also suggests that observers accepted rectangles as highly typical quadrilaterals as well.

Discussion

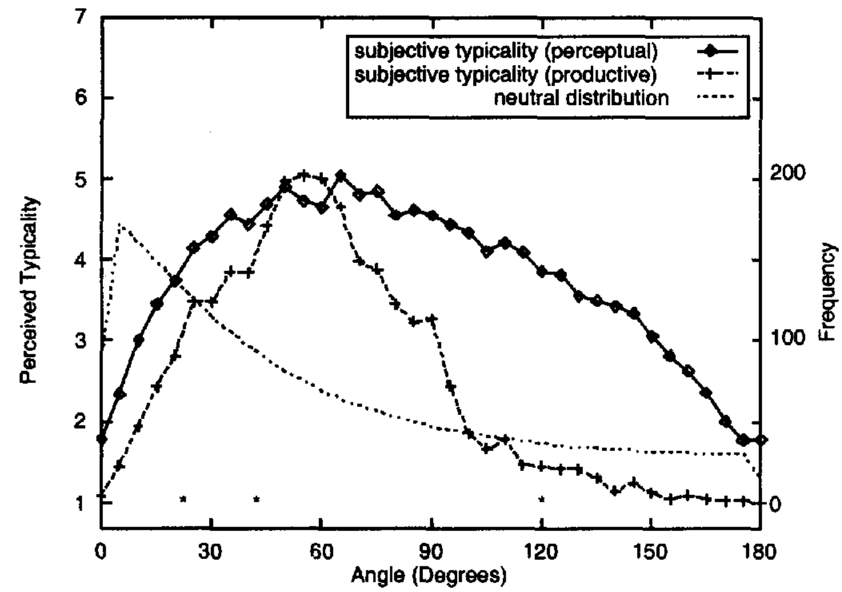
As discussed in the introduction, the canonic category distribution consists of a central mode at the prototype, with less typical cases degrading into the periphery. This is exactly the pattern observed here: For both triangles and quadrilaterals, subjective distributions showed a clear mode at a maximally typical case and a diminution of typicality at eccentric cases. The tailoff was generally sharper in the productive case, but the qualitative pattern was the same.

In several cases, the bias entailed by the mental mode was made especially clear by comparison with the neutral mode, which fell at a different location. In other cases (e.g., the two quadrilateral tapers), the mental and neutral modes were nearly the same. It is possible that these parameters are less psychologically salient than the others—less constitutive of phenomenal shape—and hence observers chose their values with something closer to an unbiased random process. By contrast, the more psychologically salient triangle angle (θ) and quadrilateral skew (α) showed clear biases away from the neutral modes.

I should note here that the scientific utility of creating neutral distributions is reflected in both the positive discovery of mental bias as well as in the rejection of spurious biases. Without the neutral distribution, one might be tempted to see the highly modal taper distributions as reflecting human preferences, but with the neutral distribu-

[5] Admittedly, a more systematic study of the effects of different sample shapes would be required to corroborate this claim. A similar remark applies to the quadrilateral data below.

A



B

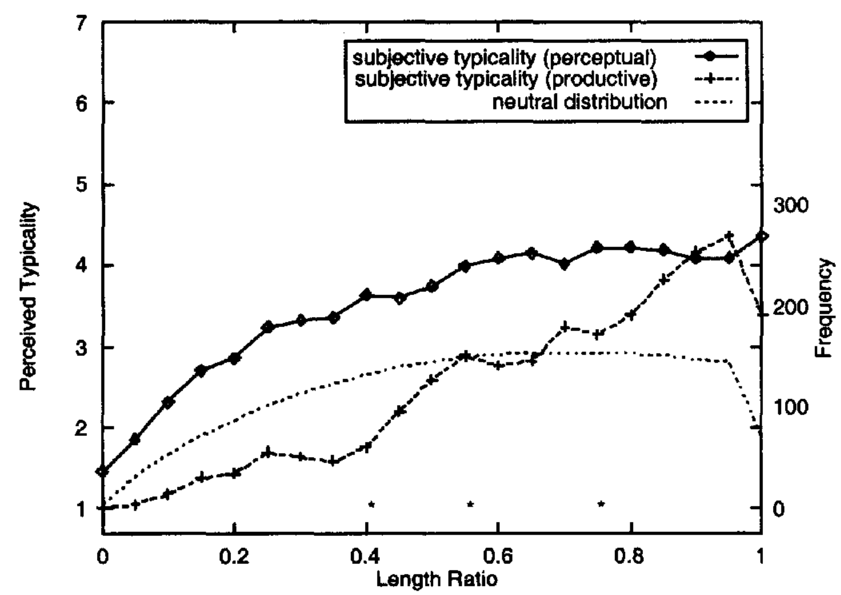


Figure 4. Results for triangles: angles (A) and length ratios (B). Each plot shows the perceptual subjective typicality distribution (from Experiment 1), the productive subjective typicality distribution (from Experiment 2), and the mathematically neutral distribution. Parameters of the sample triangle used in Experiment 2 are indicated by asterisks.

tions for comparison, it can be seen that a simple random process would produce the same general pattern.

What is especially remarkable in these results is that in both the triangle and quadrilateral cases, the psychologically modal shape was a highly regular or symmetrical form—in fact, in both cases, the *maximally* symmetrical form. As discussed earlier, from a mathematical point of view, both triangle and quadrilateral spaces are almost completely dominated by *ir*regular or generic forms. These data show that, by contrast, mental shape distributions were conspicuously biased toward regularity and symmetry.

The equilateral triangle and the square are in a sense the most regular forms possible within their respective classes, possessing the full symmetry of the so-called dihedral group, D_n for $n = 3$ and 4, respectively, and hence exhibiting a variety of mirror and rotational symmetries (see Leyton, 1992). Garner (1970) has argued that perceptual "goodness" is directly linked to the number of symmetries exhibited by the pattern, and Palmer and Hemenway (1978) have shown that observers' speed in perceiving bilateral symmetry is itself dependent on the number of symmetries exhibited by the shape. Three equidistant dots—the equilateral case—

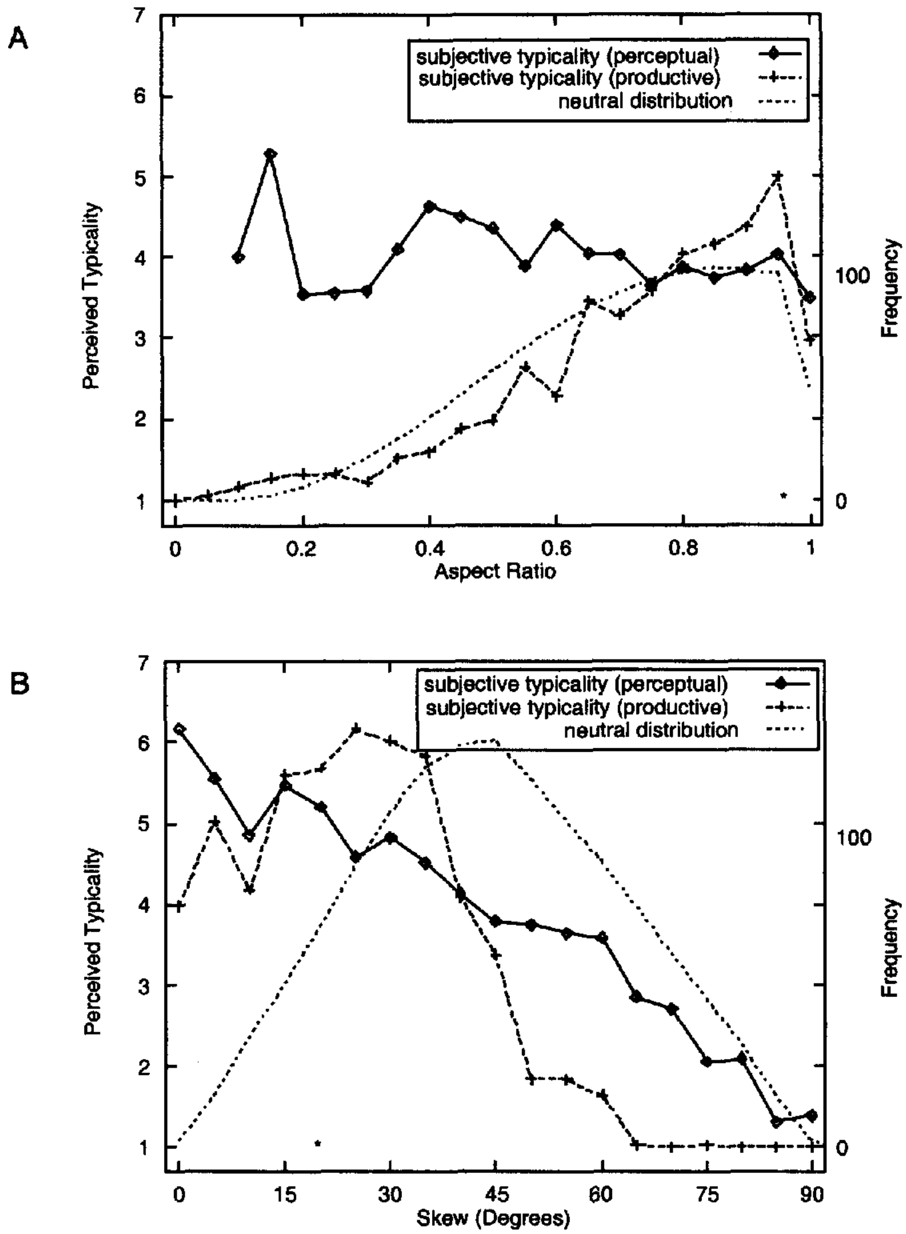


Figure 5. Results for quadrilaterals for aspect ratio (A) and skew (B). Each plot shows the perceptual subjective typicality distribution (from Experiment 1), the productive subjective typicality distribution (from Experiment 2), and the mathematically neutral distribution. Parameters of the sample shape used in Experiment 2 are indicated by asterisks.

have been shown by independent methods to be the psychologically modal configuration of three dots in the context of perceptual grouping (Feldman, 1996). The rhombus (here seen as a possible secondary quadrilateral model) is only slightly less symmetrical than the square, exhibiting 180° rotational symmetry as well as skew symmetry about two different axes (a property also known to be psychologically important; see Wagemans, 1993). Moreover, the rhombus is the shape of a square slanted in depth under orthographic projection.[6] The data suggest that these highly regular forms, rather than being regarded as atypical, are treated as prototypes for their entire classes, with other shapes in turn being treated as distortions of the canonical form (Leyton, 1984).

Modeling the data. Mathematically, one natural way to model the shape distributions is as the sum of a fixed mean

[6] Unlike quadrilaterals, all triangles are projectively equivalent, meaning that there is no unique analog of this for triangles.

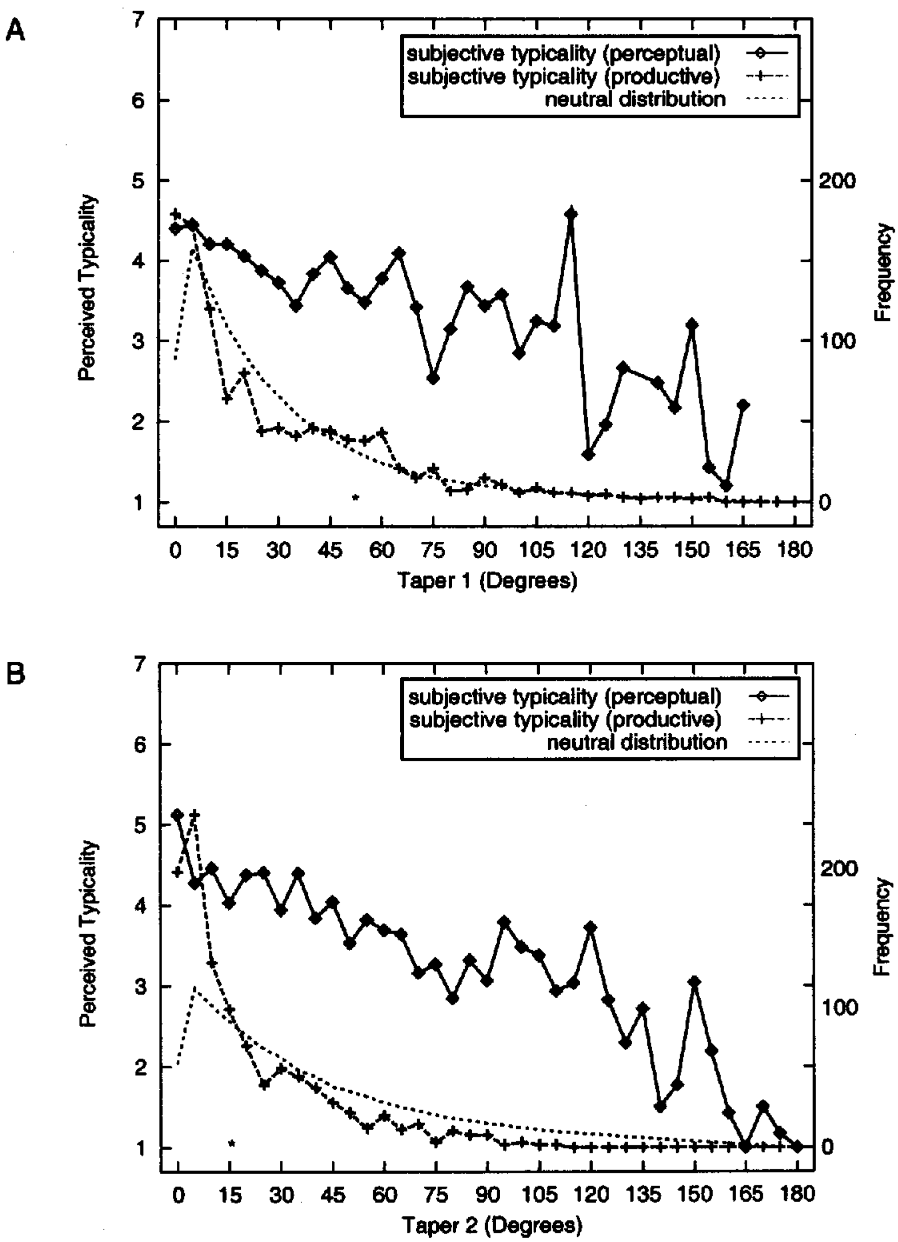


Figure 6. Results for quadrilaterals for Taper 1 (A) and Taper 2 (B). Each plot shows the perceptual subjective typicality distribution (from Experiment 1), the productive subjective typicality distribution (from Experiment 2), and the mathematically neutral distribution. Parameters of the sample shape used in Experiment 2 are indicated by asterisks.

plus an error term:

$$p(x|\text{triangle}) + \mu_T + \mathbf{e}_{(\theta,R)},$$

$$p(x|\text{quadrilateral}) = \mu_Q + \mathbf{e}_{(S,\alpha,\tau_1,\tau_2)},$$

where μ_T is the equilateral triangle, μ_Q is the square, and $\mathbf{e}_{(\theta,R)}$ and $\mathbf{e}_{(S,\alpha,\tau_1,\tau_2)}$ are each randomly distributed error vectors with mean $\mathbf{0}$. These equations give mathematical concreteness to the idea of a central prototype plus a monotonically degrading typicality function. Based on the data above, this model is at least approximately descriptively accurate. Moreover, the model is attractive in that it draws attention to the error distributions, which must be investigated in more detail to fully understand the shape distributions. In particular, the possibility of correlations among the various shape parameters must be explored; this analysis is discussed below.

From an explanatory point of view, however, this model leaves something to be desired. First, by designating μ_i a free

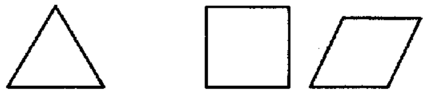


Figure 7. The psychologically modal triangle and quadrilaterals.

parameter, it gives the impression that the mode of the distribution may be placed freely at any point of the space (e.g., at an arbitrary observed object). Although this is, of course, conceptually possible, and there is no doubt that category structure can be influenced by an observed example, there is actually substantial evidence that category prototypes cannot be arbitrarily placed at empirical observations; rather, they gravitate more or less inexorably toward prototypes that are defined in some way inherent to the space, following some kind of internal mental bias. For example, Posner and Keele (1968) found that observers trained on dot patterns were better able to classify highly typical prototypes than *a*typical training examples. Similarly, Feldman (1997b) found that observers trained on different single examples within the same shape space tended to induce similar category distributions—always centered at highly symmetrical forms—with only modest regard for which training example they actually observed.

A second and related point is that even if μ_i is forced to be the maximally symmetrical point, there is no guarantee that probability elsewhere in the distribution will directly reflect intrinsic properties of the shape, simply because the error distribution is also free to take an arbitrary form.

For these reasons it is desirable to create a model in which the symmetry structure of the space plays an inherent role in the shape of the distribution. One simple way of doing this is to make probability proportional to symmetry, such as the following:

$$p(x|C_i) = k_C\sigma(x),$$

where $\sigma(x)$ is some measure of the degree of symmetry of the shape x and k_C is a normalizing coefficient that makes the distribution sum to unity. Here the judged typicality of shapes directly reflects their symmetry, forcing the modal shape to be the most symmetrical rather than leaving it as free parameter. The function $\sigma(\cdot)$ might be specified any number of ways and here simply stands in for the well-documented capacity of the human visual system to detect and rate the degree of visual symmetry (Barlow & Reeves, 1979; Dakin & Hess, 1997; Locher & Wagemans, 1993; Wagemans, 1995).

The two models may mimic each other arbitrarily closely if (as is apparently the case) each μ_i turns out to be the maximally symmetrical form in the space, that is, the maximum of the function $\sigma(\cdot)$. Hence, it is doubtful whether data such as those in this article can distinguish these models empirically. As discussed earlier, though, theoretical considerations seem to favor the latter model.

Second-order analysis. As suggested before, to understand the structure of the distributions in more detail, one needs to consider the possibility of correlations among the shape parameters. The first-order analysis above indicates where the modes in these parameters fall when the parameters are taken one at a time; I now consider how the parameters relate when considered together. Specifically, I consider the *joint densities* of the various shape parameters.

The most revealing way to do this from the data at hand is simply to plot the perceived typicality function as a function of each pair of shape parameters (perceptual task in Experiment 1) and scatter plot the shapes produced as a function of each pair of parameters (production task in Experiment 2). (Joint densities of three and four quadrilateral parameters taken at a time are perfectly well defined but impossible to plot in two dimensions.) Furthermore, numerical correlations among the various pairs of shape parameters can be calculated.

Triangles. Figure 8 plots perceived typicality as a function of the two triangle parameters (Experiment 1). The mode at $\theta = 60°$, $R = 1$ (i.e., the equilateral triangle) is clearly visible. Figure 9 is a scatter plot of the triangles produced by observers (Experiment 2); again, the mode at equilateral is evident.

In the production task, the correlation between R and θ can be calculated directly from the data; its value is $-.2234$. In the perceptual case, the correlation can be asymptotically estimated (by creating a set of observations including n duplications for a rated score of n), yielding an estimate of $-.1726$. By Fisher's exact test, the critical value of r with $p = .01$ is .0848, meaning that both these correlations are significant at the .01 level but are not significantly different from each other.

Quadrilaterals. For quadrilaterals, there are four parameters and hence six pairs. Figure 10 plots typicality as a function of each pair (Experiment 1), and Figure 11 shows the distribution of shape as a function of each pair (Experi-

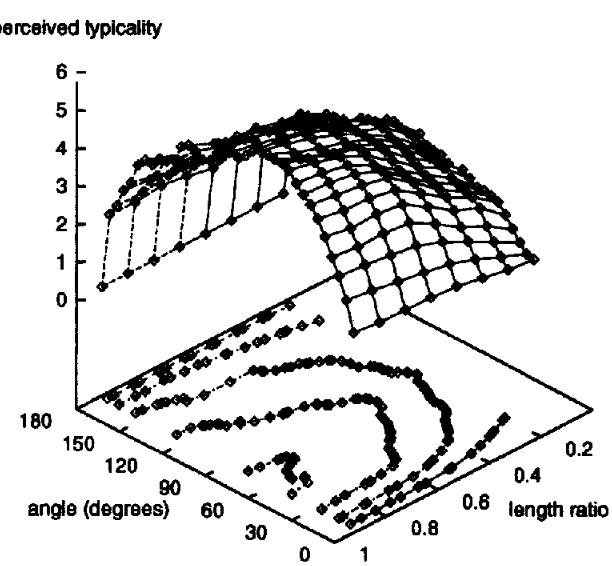


Figure 8. Surface and contour plot of observers' typicality ratings (perceptual task in Experiment 1) plotted as a function of angle and length ratio. The mode at equilateral is clearly visible.

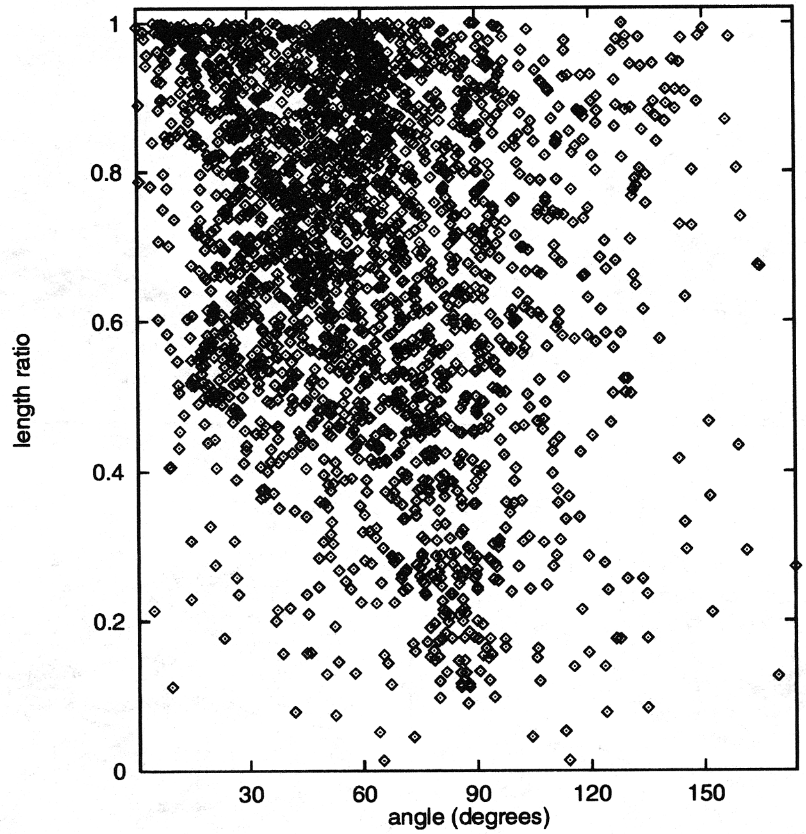


Figure 9. Scatter plot of the triangles drawn by observers (production task in Experiment 2) showing density in angle/length-ratio space. The mode at equilateral is clearly visible.

ment 2). Again, the mode at square is clearly visible in each plot, especially in the production data.

Tables 1 and 2 show the correlation matrices for the four quadrilateral parameters taken from the perceptual and production data, respectively. The critical value of r at the .01 level is .0723. Again, this is both the threshold of significance for each individual correlation as well as the threshold of significance for the difference between corresponding correlations in the two tables. Correlations of $\alpha \times \tau_1$, $S \times \tau_2$, $\alpha \times \tau_2$, and $\tau_1 \times \tau_2$ are significantly different in the two tables.

Generally, because of the large sample sizes here, significance levels of small correlations may be overestimated. Nevertheless, it is clear that several of the pairs of parameters are highly correlated, and thus the error distribution is far from being jointly independent. Explanations for the particular pattern of dependency is admittedly speculative, but a few interesting patterns do suggest themselves.

One conspicuous pattern is that correlations are generally higher in the production data than in the perceptual data. For example, in the production data there are high correlations between skew α and each of the two tapers τ_1 and τ_2. Looking at the scatter plots (see Figures 11d and 11e), one can see clear modes along the horizontal axis and along the diagonal. The horizontal axis (taper = 0°) corresponds to parallelograms (i.e., quadrilaterals with skew but no taper), another highly regular form. The diagonal (approximately the line defined by the equation $\tau = 2\alpha$) also corresponds to a highly regular subtype: right trapezoids, in which one taper is automatically twice the magnitude of the skew (see Figure 12). This is especially intriguing in light of the role virtual trapezoids are thought to play in symmetry detection (Wagemans, Van Gool, Swinnen, & Van Horebeek, 1993).

In summary, some of the correlations in the production data appear to be the result of elevated probabilities of certain special subtypes. Such special subcategories in a shape class, exhibiting a variety of symmetries and regularities, can be shown to be important in the mental representation of shape categories (Feldman, 1997b). Here they play a role analogous to but secondary to the central maximally symmetrical mode: Just as the overall mental shape space is biased toward the maximally symmetrical shape, it is also biased, albeit to a lesser degree, toward other regular modes.

A Bias Toward Regular Form

The main conclusion of this article is that mental models of triangles and quadrilaterals show a marked bias toward regular forms. Shape categories in effect consist of a central, highly symmetrical prototype plus a degradation term that increases monotonically with distance from the prototype. This result is, of course, highly reminiscent of conventional

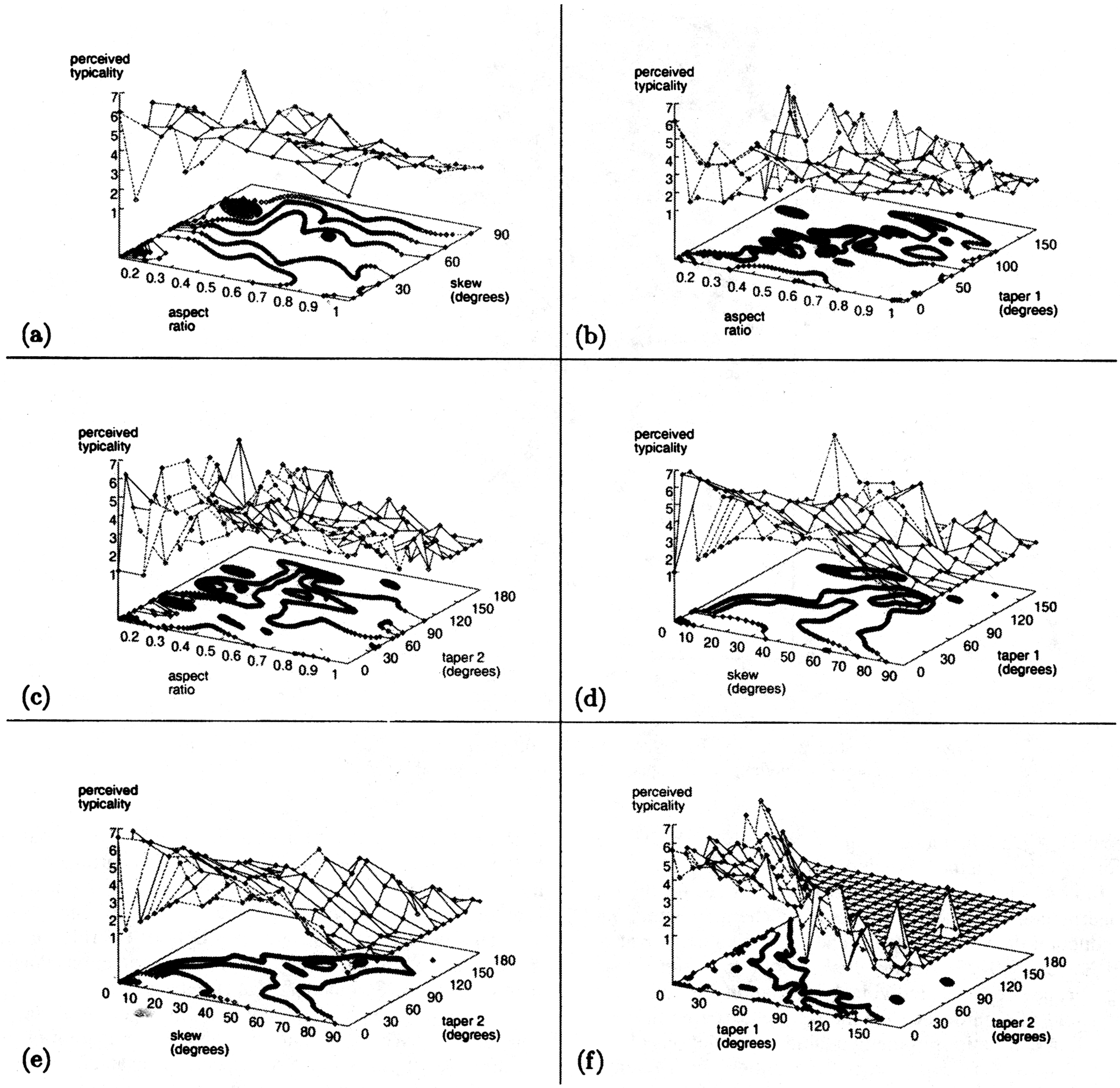


Figure 10. Surface and contour plots of observers' typicality ratings (perceptual task in Experiment 1) plotted as a function of each pair of the six dimensions: skew versus aspect ratio (a), Taper 1 versus aspect ratio (b), Taper 2 versus aspect ratio (c), Taper 1 versus skew (d), Taper 2 versus skew (e), and Taper 2 versus Taper 1 (f).

views of category structure in cognitive categories such as "bird," which are often regarded as consisting of highly typical prototypes surrounded by more unusual cases whose typicality degrades monotonically with dissimilarity from the prototype (Smith & Medin, 1981). What is remarkable in the current case is the greater degree of mathematical transparency of the underlying shape spaces, allowing a precise characterization of the mental bias, and the fact that the prototypes turned out to be extremely special rather than mathematically typical cases.

The preference for regular forms in perception has been widely noted (Hochberg & McAlister, 1953; Kanizsa, 1979), although its role is subject to debate (Hatfield & Epstein, 1985; Perkins, 1976). In the course of experiments on slant perception in infants, Slater and Morison (1985) found that newborn infants already showed a preference for squares—a

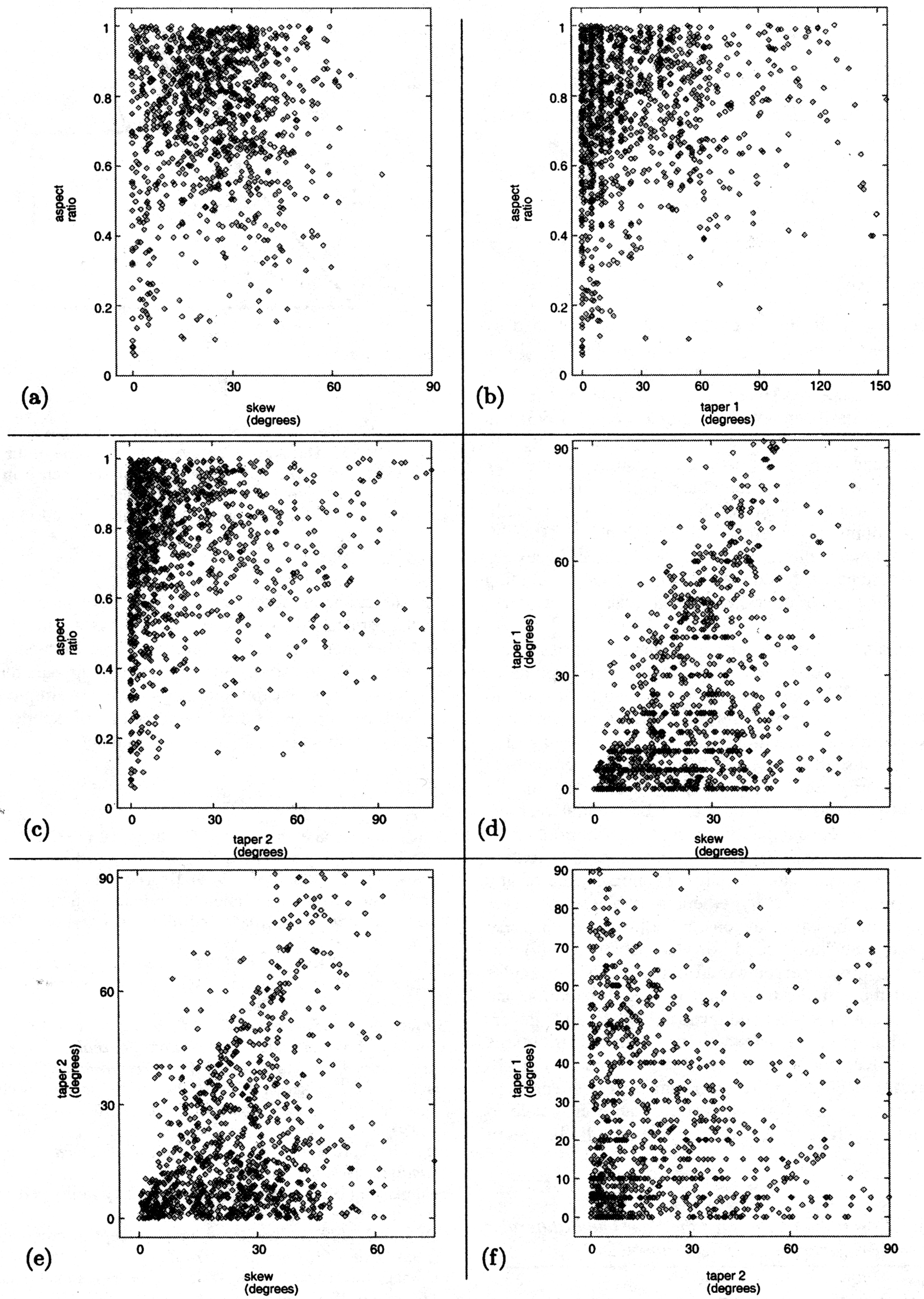


Figure 11. Scatter plots of the quadrilaterals drawn by observers (production task in Experiment 2) showing each pair of the six dimensions: skew versus aspect ratio (a), Taper 1 versus aspect ratio (b), Taper 2 versus aspect ratio (c), skew versus Taper 1 (d), skew versus Taper 2 (e), and Taper 2 versus Taper 1 (f).

Table 1
Correlation Matrix for the Four Quadrilateral Parameters (Perceptual Task in Experiment 1)

Parameter	1	2	3	4
1. S	—			
2. α	.0658	—		
3. τ_1	.1829*	−.0548	—	
4. τ_2	−.0960*	.1756*	−.2130*	—

*$p < .01$.

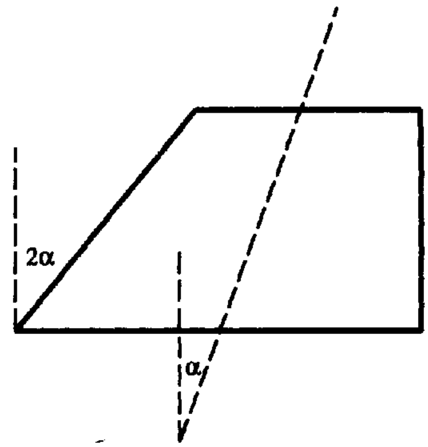


Figure 12. A right trapezoid whose taper is twice the magnitude of its skew (α). The skew is the average orientation of the left and right sides (measured here from vertical), and hence in a right trapezoid it is automatically half the value of the taper (the difference in orientation between the left and right sides).

preference so strong that it could not be extinguished by familiarity. In the completion of occluded figures, global symmetry of the completed figure has been shown to override local properties such as good continuation (Sekuler et al., 1994). It has been noted (Wagemans et al., 1993) that human regularity detection is at least global enough to operate on patterns of four dots (i.e., virtual quadrilaterals). Since the gestaltists, the preference for the most regular interpretation has continued to play a central role in mathematical accounts of perception (Buffart et al., 1981; Feldman, 1997a; Leeuwenberg, 1971; Van der Helm & Leeuwenberg, 1991; Richards, Jepson, & Feldman, 1996), including in the specific case of shape categorization (Feldman, 1997b).

A customary objection to simplicity-based theories is that the regular forms observed in psychological hypotheses simply reflect a lifetime of sensory observation of a world dominated by regular forms. If these preferences themselves guide perceptual organization and grouping, as suggested above and as usually assumed, then this view contains a vicious circularity. Before any grouping or organization, the visual field contains a riot of localizable spatial features arranged in an approximately random distribution, as is evident when one inspects the output of a simple feature detector on a typical natural image. In the absence of any grouping procedure, the hypothetical unbiased observer would, of course, have to consider all subsets of these feature points, not just "good" or well-behaved configurations. Hence, this observer would expect to find triangles and quadrilaterals in the world in about the distribution one would expect from triplets and quadruplets of points generated at random (i.e., almost exactly the neutral distributions exhibited above, not the subjective distributions). Put another way, for an observer to collect statistical frequencies of various shapes in the world, it would require an oracle to indicate just which parts of the raw image actually belong to the same shape, rather than using perceptual grouping mechanisms for this purpose, as is always assumed—and it could not then turn around and use these frequencies post hoc to help decide how to group. Hence, it appears that these subjective shape distributions originate not in simple empirical observation but rather in a nexus of subtle mental stereotypes about regularity of form and pattern.

Table 2
Asymptotic Correlation Matrix for the Four Quadrilateral Parameters (Production Task in Experiment 2)

Parameter	1	2	3	4
1. S	—			
2. α	.0639	—		
3. τ_1	.1289*	.4776*	—	
4. τ_2	.0952*	.4209*	.1232*	—

*$p < .01$.

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