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Mapping the mental space of rectangles

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Abstract. The cognitive structure of a shape space—the space of rectangles—is explored by a nonmetric scaling technique. Our experiment was designed to extract the major transformational paths or 'modes' that characterize the mental shape space. Earlier studies of rectangle similarities using multidimensional scaling have provided conflicting evidence about whether the coordinate system of the mental rectangle space is based on height and width or on area and shape (ie aspect ratio). Our study reveals shape to be the single dominant factor. We suspected that earlier evidence for a height–width parameterization might have been due to the presentation of rectangles upright in a pseudo-gravitational coordinate system (whereas our rectangles are randomly rotated). In a control experiment with upright (vertical or horizontal) rectangles, the heavy bias towards shape preservation was still the dominant mode. In addition, however, a secondary bias towards change of height or width emerged, exactly following the pattern expected from the biasing change in context. This finding established a concrete path by which context and frame can influence the way shape is represented. The relevance of these findings to the cognitive organization of more complex shape spaces is discussed.

1 'Natural' transformations in rectangle space

Shape can be represented in any number of different ways: by features on the object's contour, by the geometry of its parts, or even by the raw set of image points it occupies. Many treatments of shape (eg Leyton 1992) emphasize transformations by which shapes are deformed into other shapes. For example, a parallelogram can be regarded as a skewed rectangle, which itself would be regarded as a stretched square, and so forth. In this view, a shape is described in terms of the transformations (eg of some primitive object) that gave rise to it. For arbitrary shape spaces, one can imagine a wide variety of possible transformations and primitive objects. Mathematically, a transformation is just a path through the space, an infinite number of which pass through any space. Hence if shape description is pegged to transformations, any shape can be described in any number of different ways.

Nevertheless, some transformations seem more intuitive or reasonable than others. The choice may be due to mathematical simplicity, or to some physical analogy (eg dilation, taper, skew, etc). When considering what aspect of shape should be changed, one must at the same time ask the complementary question: what should be preserved (eg area, length, etc)? Again, certain choices seem more intuitive than others. In this paper we investigate what shape transformations are regarded as 'natural' by human perceivers in a simple shape space, the space of rectangles.

Rectangles have two degrees of freedom, often taken as length (l) and width (w). However, many other parameters are possible. Two commonly used are shape ($s = w/l$) (ie aspect ratio or 'eccentricity') and area ($a = lw$). Other less obvious combinations (eg area-over-perimeter-squared, $a/[2(l+w)]^2$) are occasionally encountered as well. An area–shape coordinate system $\langle a, s\rangle$ has been favored by some authors (Krantz and

Tversky 1975), while others have argued for the $\langle l, w \rangle$ system (Borg and Leutner 1983), and still others for both (Schönemann and Lazarte 1987). All authors agree that a simple metric model of rectangle space must be supplemented either by interactions between dimensions (Krantz and Tversky 1975) or by what Borg and Leutner (1983) call "more complicated composition rules", the form of which is still unclear. One might expect that when comparing two rectangles R_1 and R_2, observers might qualitatively alter their method of comparison depending on the nature of the observed relationship. If a particularly favored or 'natural' parameter is preserved between R_1 and R_2, for example, then they might be regarded as similar, while if no particular parameter is preserved, the rectangles might be regarded as dissimilar despite having the same distance in an unbiased metric sense. However, identifying the 'natural' parameters is a challenge: given any two rectangles, one can, of course, always interpolate a smooth path, albeit perhaps an arbitrary or complex one. Hence our assumption is that human observers have only a small set of predetermined favored paths through any shape space, which are used to orient and structure comparisons.

Previous work on the structure of rectangle space has been based purely on similarity judgments (Krantz and Tversky 1975; Borg and Leutner 1983; Schönemann and Lazarte 1987), emphasizing the metric structure of the space, as revealed by multidimensional scaling (MDS: Torgerson 1952; Shepard 1962; Kruskal 1964). Moreover, most studies have presented rectangles 'upright', aligned to a horizontal-vertical coordinate system that might introduce a bias towards a particular choice of parameters (eg height and width). Our method, called trajectory mapping (TM: Richards and Koenderink 1995), differs from these previous scaling studies by using a technique that explicitly encourages the observer to reveal how he or she believes one shape can be transformed into another. Hence we more directly test for the parameterizations that take R_1 into R_2, probing what parameters subjects regard as the 'natural' paths through the space. In addition, we also used randomly oriented rectangles to minimize framing context effects.

2 Trajectory mapping (TM)

Our objective was to find the shape transformations taking one rectangle, R_1, into another, R_2, and smoothly on into a third, R_3. Triplets of rectangles are needed in order to create smooth paths through the space, not just the pairs that are commonplace in MDS approaches, because when taking pairs in isolation there is no guarantee that the transformation from R_1 to R_2 is related to that from R_2 to R_3. This suggests an analogical approach [akin to that of Rumelhart and Abrahamson (1973)] in which R_1 is to R_2 as R_2 is to R_3.(1) We can also take the inverse transformation taking R_1 into some 'previous' rectangle in the implied sequence. This is the essence of the trajectory mapping technique that we used.

The method we used was as follows: on each trial, the subject is shown two reference examples, the two fixed rectangles R_1 and R_2. The subject is then asked to consider the 'trajectory' between R_1 and R_2. That is, if one regards the right example as the result of somehow changing or transforming the left example, what aspect is actually changing and what is held constant? If this transformation were extrapolated a bit further, what would the next example in the trajectory look like? This is the *right extrapolant*. Likewise, the subject also indicates a *left extrapolant*, the result of extrapolating the trajectory in the other direction. Similarly, the subject chooses an *interpolant*, an example judged to fall midway between the two examples along the interpreted trajectory. The result is a complete trajectory of five items: x, R_1, y, R_2, z, where x, y, and z are the three new examples chosen by the subject. The quintuplet of examples from each trial thus represents what the subject regards as a natural, coherent trajectory through the

(1) Rumelhart and Abrahamson proposed a parallelogram rule: A is to B as C is to D. For our study we are interested in continuous paths, so we equate B and C, forming triplets.

mental space in question—like a local piece of a flow field. In most previous studies, subjects chose x, y, and z from a fixed finite list of examples. In the variant technique used below, subjects are actually able to create novel shapes with the computer mouse, which is more appropriate when the underlying space is continuous. Ideally, the result of this technique will be a map of the topology of the trajectories or 'flow patterns' in the shape space, as opposed to the metric structure as would normally be revealed by MDS approaches.

In addition, two other options are given to subjects. We instructed subjects to classify pairs as "incomparable", whenever they felt that *no* reasonable transformation could actually carry R_1 into R_2. Similarly, subjects could choose a "boundary" option instead of one or both of the extrapolants (x or z), whenever they felt that the trajectory actually ended at R_1 or R_2 rather than continuing. These additional options allow us to avoid assuming that all rectangles are commensurable in a uniform transformation space or similarity function. These options also distinguish our method from earlier scaling techniques such as that of Carroll and Chang (1972). For a complete description of the method, along with an objective evaluation function, see Gilbert (1997).

In our methodology, subjects are asked to extrapolate (or interpolate) from two shapes to a third. As discussed above, there are in principle many ways that they might choose to do this. We assume that, to the extent possible, they will attempt to preserve salient features that are shared by the reference shapes. Shared features will be carried over to the extrapolant (or interpolant), while other features will be allowed to vary. Hence examination of what features are preserved will reveal what features subjects regard as salient or dominant in the shape representation. Of course, other assumptions about the logic of subjects' extrapolations are possible. This issue is discussed more below.

The result of the TM procedure is a set of quintuplets, each of which represents a piece of a trajectory through the feature space—a local piece of 'flow' through the flow field. Again, each trajectory represents some type of rectangle transformation: perhaps a change in size, aspect ratio, length, or width; or perhaps some combination of them. The exact type of transformation depends on the local *orientation* of the flow through the space (see below). Typically, certain directions are more frequent in the data than others. That is, there are certain favored transformational paths—the major 'highways' through the space. We call these favored paths *modes*.

Figure 1 illustrates how the presence of a mode might influence the comparison of nearby rectangles.[2] In the figure, the path from y (the interpolant) through R_2 (the right example) might continue straight through the space—ie a simple continuation of the direction joining the examples R_1 and R_2 (again, à la Rumelhart and Abrahamson). This extrapolation is the most reasonable one, given an unbiased metric model of the space. Alternatively, an extrapolation guided by a nearby mode will land at z'—extrapolation lies in a direction parallel to the mode. A parallel argument applies to the leftward extrapolation to x (not shown in the figure). These two directions of extrapolation—the straight one, based only on the reference positions, and the modal one, based only on the direction of the mode—represent extremes; in practice observers may choose some combination or compromise between these alternatives. Hence the influence of modes is reflected in the data in that measured extrapolations fall more frequently in certain directions than others, especially as compared to the population of straight extrapolation directions. Hence the main analysis is simply a frequency

(2) In the figure, we have drawn x, R_1, y, R_2, and z at equal intervals. Alternatively, one might expect z (and x) to fall further out, under the assumption that the extrapolated distance faithfully reflects the distance from R_1 to R_2. However, Schönemann and Lazarte (1987) have found that distances in rectangle space tend to be subadditive, which is more consistent with our picture. In any case, none of these considerations matters to our analysis, which is based only on the orientation of the various segments, not their lengths.

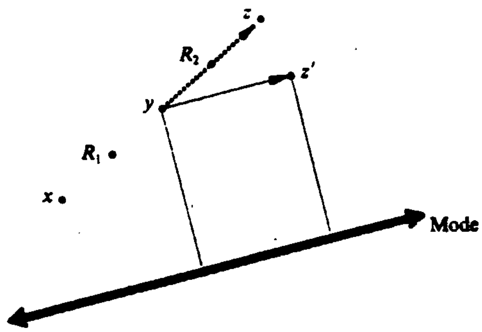


Figure 1. Possible directions of extrapolation in the presence of a mode.

plot of the extrapolation orientations, ie the direction from the interpolant to the right extrapolant and that from the interpolant to the left extrapolant (plus a differential plot in which this frequency is compared to the raw number of straight extrapolations in that direction). In addition, there is the possibility that the pattern of modes might be different in different parts of the space—eg certain kinds of extrapolations are influenced by one mode, and others by a different mode. Hence a secondary analysis is to plot the frequency of orientations as a function of the fixed example direction.

It should be remarked that, because our method searches for positive evidence for modes through the space (rather than, say, setting one parameterization against another), it is entirely possible that we will find fewer modes or more modes than there are degrees of freedom in the space. This means that the dominant modes may underdetermine or overdetermine the space, and hence cannot necessarily be regarded as a parameterization of the space. Hence our data do not promise to choose a parameterization, but rather, as discussed above, to find psychologically dominant transformations.

3 Experiment 1: Trajectories in rectangle space

We report an experiment in which a version of the TM technique was used to explore a shape space—the space of scaled rectangles.

3.1 *Method*

3.1.1 *Subjects.* Six naive subjects were paid for their participation.

3.1.2 *Materials.* We chose a set of 18 rectangles, intended to cover a wide range of sizes and aspect ratios (see figure 3). There were 7 widths (ranging from 3 pixels to 198 pixels by approximate powers of two) and 4 lengths (from 24 pixels to 196 pixels by powers of two). All but 18 of the possible combinations were eliminated either because the 'width' was greater than the 'length' or because the aspect ratio was extreme. The resulting set of 18 rectangles has aspect ratios ranging from $\frac{1}{8}$ to 1 (square), including four different sizes of square. These rectangles were presented on a monitor measuring 42.5 cm diagonally at about 50 cm viewing distance.

A script was constructed by taking all 153 $\left[= \binom{18}{2}\right]$ possible pairs of the 18 rectangles, and constructing one trial from each pair, presenting the pair either left–right or right–left, determined randomly. Each rectangle was rotated by a different random amount each time it appeared, and subjects were instructed to ignore orientation completely. (As mentioned above, we suspected that presenting the rectangles at a fixed orientation would create a fixed pseudo-gravitational framework that might introduce artifacts not related to shape per se.)

3.1.3 *Procedure.* On each trial, the subject saw the two fixed rectangles R_1 and R_2, and three squares in place of *x*, *y*, and *z* which he or she could manipulate with the mouse (figure 2).

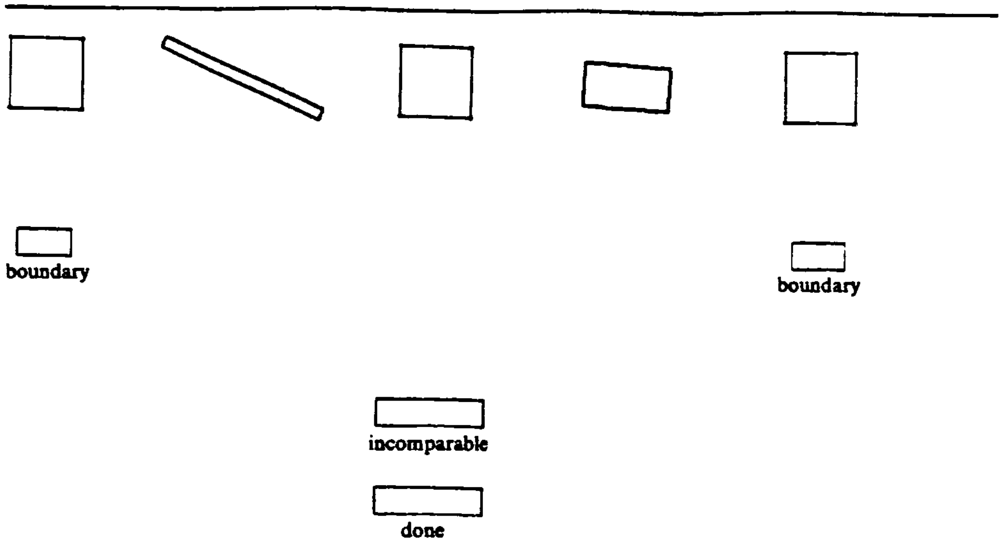


Figure 2. A typical trial screen, containing the two fixed rectangles R_1 and R_2, and three squares that subjects could manipulate with the mouse to construct the left extrapolant x, the interpolant y, and the right extrapolant z.

Subjects were instructed that each click of the mouse button would move the nearest vertex to the mouse position; repeated clicks allowed the subject to move each vertex to any location desired. Subjects were free to spend as much time as needed to get each rectangle to the desired appearance. Alternatively, they could click "boundary" under one of the outer rectangles, or "incomparable". When they were satisfied with the trajectory through the five rectangles on the screen, they clicked "done", at which time the computer recorded the final coordinates of the x, y, and z vertices, and the final disposition of the "boundary" and "incomparable" buttons.

Because the task is fairly laborious, the resulting script was considered too long for one subject to carry out. So the script was divided into three parts (of 45, 45, and 63 trials), with each subject assigned one part. The six subjects thus completed two complete traversals of the script. Notice that dividing the script up among different subjects works against our underlying hypothesis that rectangle space will manifest a coherent and uniform set of trajectories.

3.2 *Analysis*

We analyze the results in several stages. First, we establish a convenient parameterization for rectangle space, in which orientation is well defined. Then we establish that contiguous triplets from subjects' trajectories are mostly nearly collinear in this space, demonstrating that the trajectories represent approximately smooth paths through the space (smooth curves are locally well approximated by straight lines). Then, we consider the frequencies of different extrapolation orientations in this space, looking for modes (particularly frequent orientations). Finally, we consider second-order effects.

3.2.1 *A parameterization for rectangle space*

As mentioned above, the 2-D rectangle space can be parameterized in many ways. Each rectangle is completely described by a length l and a width w, such that $w \leqslant l$.[3]

[3] Note that when other authors have written h and w they have usually meant height and width as defined in an upright coordinate system, so that w might be larger than h. Because the rectangles in experiment 1 rotate freely, we write l for the length of the longer dimension and w for the length of the shorter dimension to emphasize this distinction. l and w are extracted from the vertex coordinates as follows: l is the average length of the longer pair of opposite sides, and w is the average length of the shorter pair of opposite sides.

It is convenient to use logarithmic units. First, for notational brevity we define

$$L = \ln l,$$

$$W = \ln w.$$

In addition, we create two new parameters, the logarithm of the aspect ratio

$$S = \ln \frac{w}{l},$$

and the normalized log area

$$A = \ln \frac{wl}{a_0},$$

where a_0 is the area of some standard rectangle (the choice of which is unimportant; different standards lead to rigid translations of the coordinate system right or left). Notice that the $\langle A, S \rangle$ coordinate system is simply a 45° rotation of the $\langle L, W \rangle$ system (figure 3).

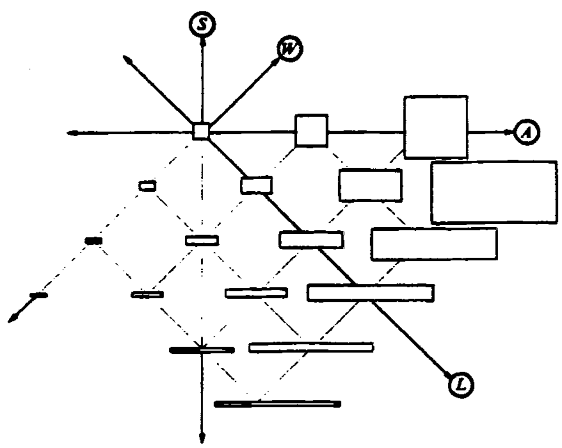


Figure 3. Rectangle space, showing how the $\langle L, W \rangle$ coordinate system is a 45° rotation of the $\langle A, S \rangle$ system. As A increases (east), area increases while shape remains constant; as S increases (north), the rectangle becomes more square, while the area remains constant; as W increases (northeast), the shorter dimension grows, while as L increases (southeast) the longer dimension grows. The rectangles shown are the ones used in the experiments.

Each trial of the experiment yielded a trajectory of (at most) 5 rectangles: x, R_1, y, R_2, and z. Trials in which the subject chose "boundary" once yielded only four rectangles (x, R_1, y, R_2; or R_1, y, R_2, z), and trials in which the subject chose boundary twice yielded only three rectangles (R_1, y, R_2). "Incomparable" trials were omitted completely.

Each trajectory constitutes a single continuous piece of the rectangle 'flow field'. We first want to confirm that these trajectories represent approximately smooth paths in $\langle A, S \rangle$ space (or, for that matter, in $\langle L, W \rangle$ space, which is a rigid rotation). We do this by confirming that most of the contiguous triplets [ie triplets of the form (x, R_1, y), (R_1, y, R_2), and (y, R_2, z)] are nearly collinear. (If they were not, it would mean either that subjects' extrapolations were largely unpredictable, or that $\langle A, S \rangle$ was an inappropriate coordinate system.) To do this we simply map each rectangle triplet to $\langle A, S \rangle$ coordinates, and then measure the interior angle of the point triplet. A frequency histogram of the resulting angles is shown in figure 4. The angles are heavily clumped near 180°. This means that the underlying trajectories (the curves that these triplets can be thought to be samples of) are, locally, nearly straight lines in our log–log representation. Of course, not all the triplets are straight—nor do we expect them to be, if some of the extrapolations are influenced by modes (consider figure 1).

This finding does not help decide between $\langle A, S \rangle$ and $\langle L, W \rangle$. Moreover, it says nothing about preferred directions through the space. Furthermore, it leaves the many

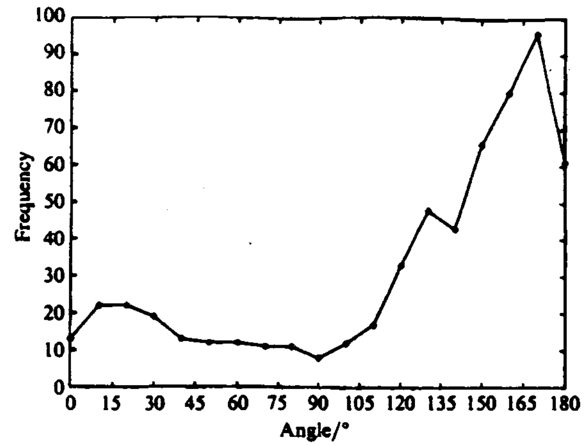


Figure 4. Frequency histogram of the interior angles of rectangle triplets drawn from subjects' trajectories.

noncollinear triplets unexplained. As suggested above, these more subtle questions—which relate to the nonmetric effects and 'complex composition rules' other observers have noted—might be explained by the presence of preferred modes in the space. In the next stage in the analysis we investigate this possibility directly.

3.2.2 *A mode in rectangle space*

We now consider the directions of extrapolation in the trajectories, ie the directions of the vectors $\overrightarrow{yx}$ and $\overrightarrow{yz}$. By using these directions we are in effect assuming that the interpolant y constitutes a central reference point from which both extrapolations emanate (an assumption supported by the collinearity of most of the triplets). Notice, moreover, that these extrapolation directions are 'pure data' unpolluted by the fixed example rectangles. All orientations are normalized to the interval [0°, 180°). A frequency histogram of the resulting orientations is shown in figure 5a.

This histogram has a clear bowed shape, with much of the probabilistic mass lying near 0° or 180° (ie along the A axis). That is, rather than extrapolate rectangles straight through the space, subjects tend to be biased towards *changing area while preserving shape.*

However, there is a subtle bias in the data here. Because the rectangle set itself consists of items at the vertices of a square grid (as in every rectangle experiment in the literature), there is an overemphasis on vertical, horizontal, and 45° diagonal motions. To compensate for this bias, we subtract the distribution of grid orientations[4] to create

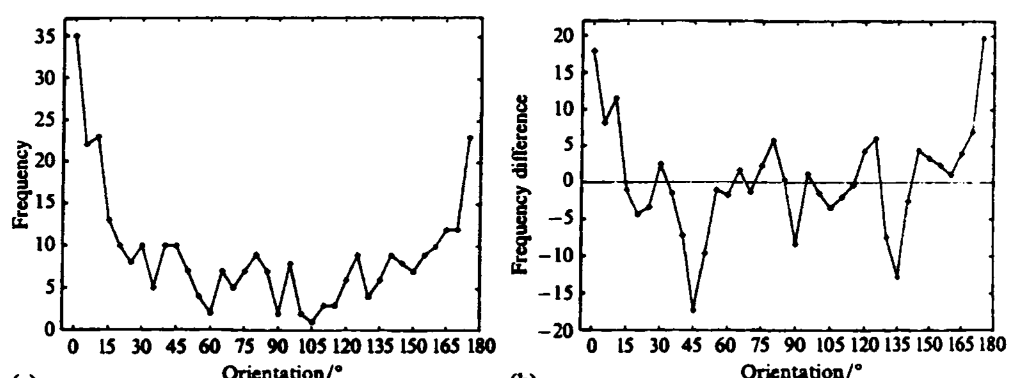


Figure 5. (a) Histogram of orientations from experiment 1 in raw form; and (b) plot of frequency differences (orientation frequency minus grid orientation frequency).

[4] This distribution was estimated from the raw grid orientations with the use of the Parzen estimator with a Gaussian kernel, and then scaled by the total number of raw orientations.

a plot of frequency differences (shown in figure 5b). This plot reflects the true direction preferences of subjects after the biasing effect of the stimulus set has been removed. In the different plot, the bias towards shape (0° and 180°) is still plainly visible. By contrast, the values at 45°, 90°, and 135° (corresponding to motions along *W*, *S*, and *L* axes, respectively) are now negative. That is, considering how frequently the fixed reference direction lay along the *W*, *S*, and *L* axes, subjects rarely extrapolated in those directions.

3.2.3 *Second-order interactions*

We now analyze a more subtle aspect of the data from experiment 1. The above plot simply reflects which orientations are favored overall. Clearly, one might expect different directions to be favored depending on the orientation of the reference rectangles in the space. For example, if the reference pair lie nearly parallel to the *A* axis, we would expect to see shape preserved. But what if they lie closer to the *L* or *W* axes? Here there might be a bias towards shape preservation, but differing in magnitude, depending on the orientation.

In figure 6 the extrapolant direction is plotted as a function of the reference orientation. For this analysis, we use only the two extrapolants and discard the interpolant (which really represents two different orientations depending on whether one regards it as emanating from the left or from the right). Note that the set of all possible extrapolation directions defines a circle, with one rectangle fixed at the center and the other at some point on the boundary. In the figure, position along this circle thus represents the reference orientation (ie, for x, the direction of the vector $\overrightarrow{R_2 R_1}$, and for z, the direction of the vector $\overrightarrow{R_1 R_2}$). (For right extrapolants, imagine that R_1 lies at the center of the circle and R_2 on the contour, or vice versa for left extrapolants.) The orientation of the line emanating from each point represents the average orientation of extrapolants resulting from that reference orientation. The length of the line indicates the number of pairs at that orientation.[5] Thus exactly radial lines represent 'straight' extrapolations through the space, while any deviation from a radial direction represents some bias.

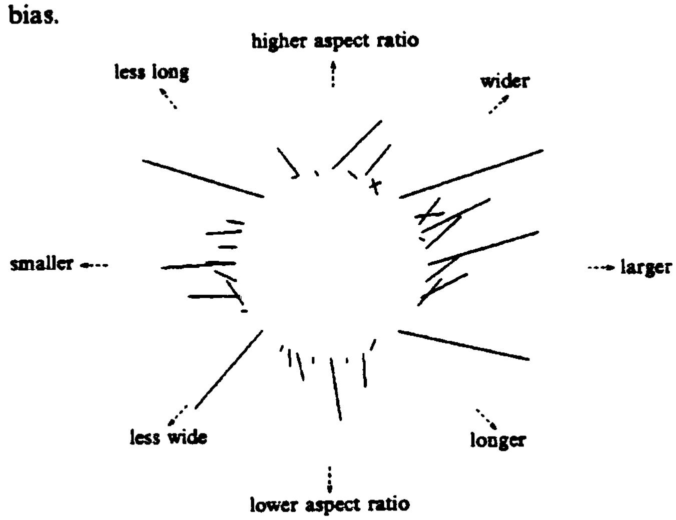


Figure 6. A 'pinwheel' showing preferred orientation as a function of the orientation of the reference rectangles. (Note that only polar direction, ie position along the interior circle, is meaningful in this plot. It is not a plot of $\langle A, S \rangle$ space.)

[5] Note that lines emanating from opposite poles of the circle sometimes have slightly unequal lengths because subjects have chosen "boundary" an unequal number of times when extrapolating to the right and to the left.

The main bias towards shape preservation found earlier is clear here: the extrapolations are clearly biased towards due east and due west, ie along the A axis, preserving shape. Even reference rectangles increasing in W or L (45° and −45°, respectively, northeast and southeast) are drawn almost completely to the shape mode.

The main exception to this trend is that changes in the direction of due south (decreasing aspect ratio) tend to be extrapolated straight. This weakly suggests the presence of a secondary size-preservation (shape-change) mode, orthogonal to the principal mode, which is only influential when the reference rectangles preserve size almost exactly. Elsewhere the 'gravitational pull' of the shape-preservation mode draws extrapolations towards the east or west.

Why is this secondary trend more conspicuous in the southerly than in the northerly direction (increasing aspect ratio)? A subtle argument may explain this trend. Reference rectangles increasing in aspect ratio are becoming more square. To subjects, squares may represent a psychologically distinct category from other rectangles (see Feldman 1997). Subjects may be reluctant to continue transformations that would cause the shape to change category. Hence they 'veer off' rather than approach the square category directly. As a result, transformations due north are extrapolated east or west, along the dominant shape mode.

4 Experiment 2: A change in context

Though direct comparisons between similarity data and our data (which do not explicitly concern similarity) should perhaps be approached with caution, there is an apparent contradiction between our finding of no preference for L or W motions and that of Borg and Leutner (1983) that height and width were the principal axes of rectangle space. We suspected that the difference might derive from our use of freely rotated rectangles, while they used upright rectangles, perhaps suggesting an $\langle H, W \rangle$ coordinate frame. Hence we ran a control experiment with the TM methodology, replicating experiment 1, but with rectangles presented in a consistently upright orientation, either in 'tall' orientation (vertical) or 'flat' orientation (horizontal).

4.1 *Method*

4.1.1 *Subjects.* Six naive subjects were paid for their participation in the 'horizontal' condition, and six in the 'vertical' condition. None of these subjects had participated in experiment 1.

4.1.2 *Materials.* Stimuli and instructions were as in experiment 1. In the horizontal condition, rectangles were presented with the longer dimension L oriented horizontally, and in the vertical condition, vertically.

4.1.3 *Procedure.* The procedure was exactly as in experiment 1. Because the method by which subjects arranged the vertices of the response rectangles was unchanged, subjects were still free to orient these rectangles freely, though they generally did not.

4.2 *Results*

The raw orientation histograms for both conditions are shown in figure 7a, and difference plots in figure 7b. Overall, the curves are very similar to those from the previous experiment, again showing the heavy bias towards the area axis (ie shape preservation) reflected in the weight towards 0° and 180°. In addition, both plots are much 'spikier' at 45°, 90°, and 135°, confirming a general biasing effect of the new upright context. In particular, there is an extremely large spike in the vertical condition at 45° (increasing width), large enough to survive in the normalized plot (figure 7b). This is exactly as might be expected from a biasing effect of the upright frame. Because the trajectory of 5 rectangles appears laterally across the screen, heights are aligned for direct comparison, but not widths (here by 'width' we mean span in the horizontal direction, which is L in the horizontal condition and W in the vertical condition). Hence similarities of height

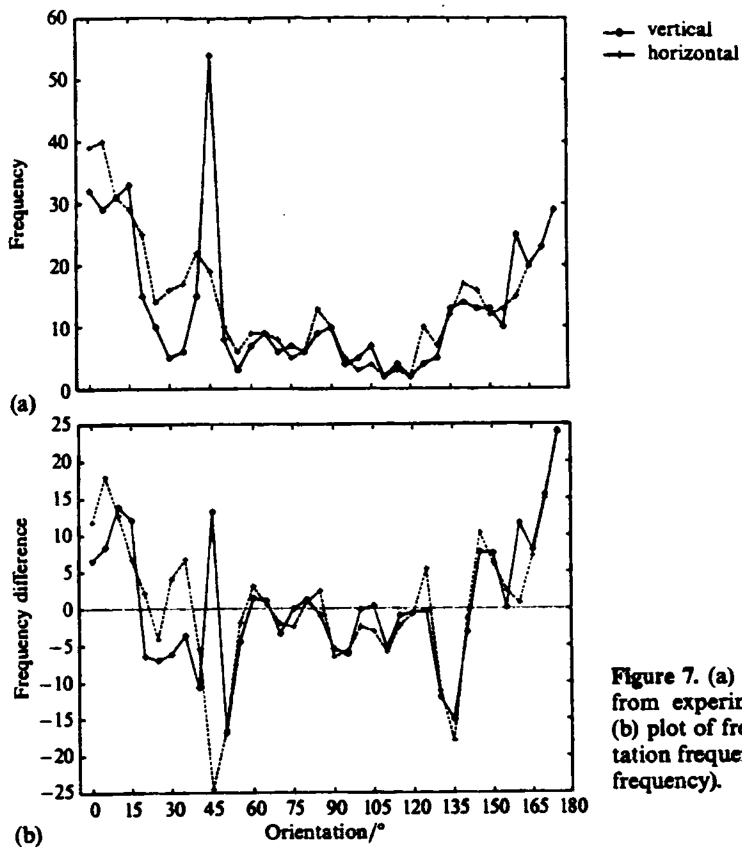


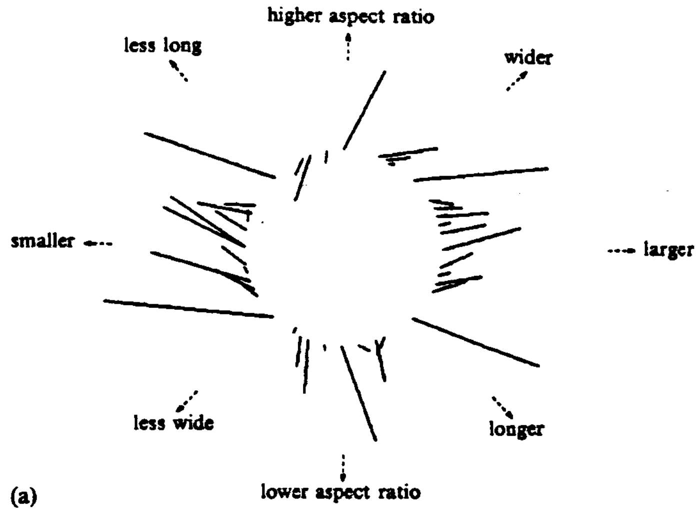
Figure 7. (a) Histogram of orientations from experiment 2 in raw form; and (b) plot of frequency differences (orientation frequency minus grid orientation frequency).

are more salient, leading to a tendency to preserve height, and transform the orthogonal parameter, width. Hence the larger mode at 45° in the vertical condition is exactly what one would expect to be the biasing effect of the upright context. Notice that the influence of the mode is exactly as portrayed in figure 1: the modal extrapolation moves along the modal direction while preserving the orthogonal parameter. This means that in the vertical condition we would expect more of a W mode (because the changing dimension is W), exactly as observed. The corresponding L mode in the horizontal condition (at 135°) is much weaker. We speculate that this is again in part due to the context, which necessarily leaves much less room for a subject to lengthen squares in the horizontal direction, as contrasted with the 'open space' in the vertical direction.

Figure 8 shows the pinwheel-style plots for the two conditions in experiment 2. Again, the main effect of shape preservation is extremely evident from the 'squashed' appearance of both plots.

5 Discussion

Most direct investigations of the structure of mental shape spaces [eg Cortese and Dyre (1996) as well as the rectangle studies cited above] have almost exclusively considered the similarity structure of such spaces, emphasizing such questions as the nature of the metric and the additivity of distances. Our results demonstrate that similarity alone does not tell the whole story. Preferred axes or modes, corresponding to distinguished properties of shape that observers attempt to preserve whenever possible, exert a salient influence. Shape modes have also been shown to play a central role in the underlying category structure of shape spaces (Richards et al 1996; Feldman 1997) as well as other perceptual spaces (Feldman 1996), and thereby in the way qualitative shape itself is represented.



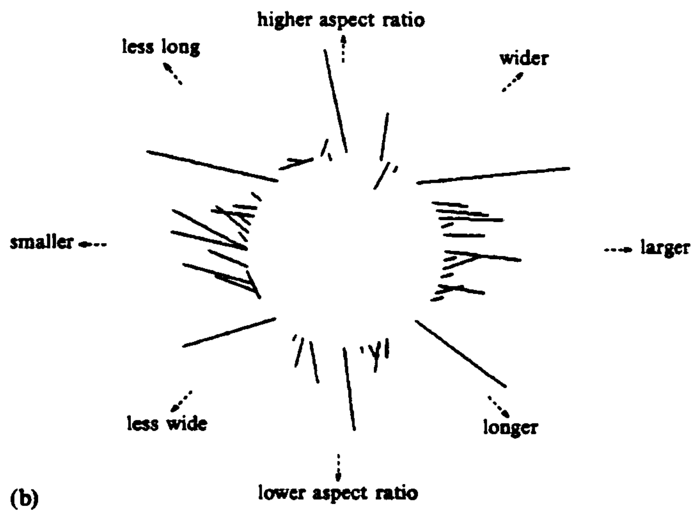


Figure 8. 'Pinwheel' plots for experiment 2: (a) vertical condition, (b) horizontal condition.

The vertical context effect discovered in experiment 2 is particularly telling. Cortese and Dyre (1996) have recently argued for a fixed choice of shape axes based on Fourier descriptors of contour.(6) Yet it is widely recognized that context and domain-specific coding rules play a salient role in structuring many kinds of cognitive spaces and mental maps. As we all know, a diamond is not a square—presumably because of the context provided by the reference frame. But perhaps less clear is that parameterization within a shape space can be sensitive to context. The adaptive value of this is obvious: context-sensitive parameterizations of a space allow the same space to be organized and reorganized depending on the useful regularities and variabilities in an ever-changing environment, giving maximal flexibility to the shape representation.

(6) Cortese and Dyre showed that judged similarity of their blob set is well modeled by two dimensions: phase and amplitude of a certain Fourier representation. Yet these were precisely the two parameters from which the blob set was constructed. Hence their subjects, pre-trained on the blob set, may well have inferred the relevant parameters of variation in this local 'context'. Hence it is unclear whether the empirical success of these two parameters in modeling their similarity judgments is evidence for their universality, as the authors claim, or of subjects' ability to choose a useful parameterization for a given set of stimuli.

The rectangle space considered here is, admittedly, a particularly simple and impoverished one, having only one true dimension of shape (ie aspect ratio). One naturally wonders about the 'natural transformations' through more realistic, higher-dimensional shape spaces. One certainly suspects that aspect ratio would not continue to be the dominant mode—just as one would not expect observers to rely on aspect ratio when comparing a rectangle to, say, a kangaroo. Rather, we suspect that the mode structure of a shape space depends on the nature of the space, with potentially large differences in preferred dimensions for different types of shape. This suspicion is greatly bolstered by our discovery of a context-dependence in the parameterization of a shape space. The high dimensionality of realistic shape spaces is, of course, an enormous practical obstacle to discovering their internal structure, accounting for the attention hitherto to such simplified spaces as rectangles. The method we have used here holds the promise of shedding light on the mental parameterizations underlying the more interesting, but much more intractable, high-dimensional spaces of natural shapes.

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