LOGICAL CAPACITY OF VERY YOUNG CHILDREN: NUMBER INVARIANCE RULES

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GELMAN, ROCHEL. Logical Capacity of Very Young Children: Number Invariance Rules. CHILD DEVELOPMENT, 1972, **43**, 75–90. *Children 3–6 years of age, when given an identification task where number was redundant to length or density, solved the task on the basis of number. Surreptitious subtraction or addition elicited strong surprise as well as search behavior whereas displacements did not. Children who noticed the change in number or length and density gave unambiguous explanations of the nature of the intervening operations and were able to indicate how to reverse the effect. These findings are taken to show young children can treat small numbers as invariant. The results are discussed in terms of why children of the same age fail to conserve number in the standard conservation task and how complex number concepts might develop.*

Studies on the ability to treat number as invariant can reveal both a logic for manipulating number and the nature of number concepts. To demonstrate that an adult treats number as invariant is to show that he has

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a logic (set of rules) that classifies the operations that can be performed on a set as relevant or irrelevant to number. The operations of addition and subtraction are classified as relevant because they change number; the operations of displacement and rearrangement as irrelevant because they do not change number. Because the operations of displacement and rearrangement produce changes in the irrelevant properties of a set (e.g., length, area, configuration), a demonstration of the use of invariance rules provides evidence with which to assess the conception of number. If the judgment of number for sets of X (when X is a cardinal number) is not altered by these changes, we may conclude that the person's concept of number does not depend on any of these properties and that the sets of X that include variations in these properties are treated as belonging to the same cardinal number class.

The standard test of a child's ability to treat number as invariant is the Piagetian conservation task (Piaget 1952). Here a child is first shown two identical sets of equal number (e.g., two rows of six chips placed in one to one correspondence) and asked if both sets contain the same number. Children initially judge the sets as equal. Then, *while the child watches,* one set is spatially transformed (e.g., lengthened). The child is once again asked to judge if both sets contain the same number and if not, which has more. Finally, he is asked to explain his judgment. Lengthening of one of the two equal sets leads children younger than 6–7 years of age to assert one set has more. Thus, in this experimental setting, young children appear to treat spatial displacement as an operation that is relevant rather than irrelevant to number. On the basis of such findings, Piaget concludes that young children do not have the logic required to treat number as invariant. He further concludes that children do not have an adequate concept of number, that is, they do not conceive of number as a dimension that is independent of the length and density dimensions.

Some investigators have suggested that failure on the number conservation task may derive from extralogical difficulties. Thus, it has been suggested that the child may not understand the words (e.g., more, less) in the questions about quantity (Braine 1959; Green & Laxon 1970; Zimiles 1963). Or, it has been argued that the child attends to irrelevant or misleading cues, a tendency that may be exaggerated by watching the transformation (Bruner, Olver, Greenfield, et al. 1966; Gelman 1969a; Mehler & Bever 1967; Wallach, Wall, & Anderson 1967). We have also suggested that the young child may still have to develop quantity-estimation procedures that lead to confident estimates (Gelman 1969b) If so, he may be quick to change estimates after a transformation, as might an adult who is not confident about an initial estimate of number.

These considerations point to the importance of developing a number invariance test that differs from Piaget's. Such a test should minimize the estimation problem given to the child, control for the use of words such as "same," "more," or "less," and reduce the probability of a child attending to irrelevant cues.

The proceedings of a magic show appear to meet these criteria. Magicians often perform tricks that involve violating the audience's invariance rules to produce mystification and surprise. Achenbach (1969) and Charlesworth (1964, 1969) have used surprise reactions to study liquid conservation and object constancy, respectively. How might the magic show be used to study number invariance rules and control for the variables outlined above? The magician first establishes the audience's expectancy for number. Then he *surreptitiously* changes the number and waits for the audience to notice the change and express surprise. If the magician happens to displace the objects the change is expected to go unnoticed, or to be explained in terms of events that could readily have occurred (e.g., "the objects slipped") and that do not matter because they do not change number. These differential reactions reveal the audience's capacity to classify correctly operations that are relevant and irrelevant to number. The magician does not use quantity words, nor does he show the operations that might direct attention away from number. These considerations lead us to predict that if we use the magician's "show" with small numbers (to control for estimation problems), young children may reveal a capacity to classify correctly the operations that are relevant and irrelevant to number.

The experiments here study children's reactions to unexpected subtractions and additions versus their reactions to unexpected displacements.

EXPERIMENT 1: SUBTRACTION VERSUS DISPLACEMENT

SUBJECTS

The Ss were 96 children (46 boys, 50 girls) attending two nursery schools and one kindergarten in the Philadelphia area and were predominantly from the middle class. There were three age groups with 32 children in each. The age ranges of the groups were 3-0 to 3-11, 4-0 to 4-11, and 5-0 to 6-5 years with medians of 3-6, 4-7, and 5-10 years, respectively.

PROCEDURE AND MATERIALS

The experiment was run in three phases over 2 days. In Phase I (day 1) *E* played games with each child individually for approximately 15 minutes in order to establish rapport. Phase II (on day 2) served to establish expectancies about two arrays of mice. Phase III followed immediately after Phase II and served to assess how the children treated surreptitious subtractions or displacements in the sets. Phases II and III together lasted approximately 20 minutes.

The toys used on day 1 were chosen with the age of the child in mind.

The materials used for Phases II and III consisted of a set of green toy mice (1.2 × 3.2 cm) on two white plates (14 cm diameter), some Velcro to hold the mice in position on the plates, and two white cans (17.5 cm high × 15 cm diameter).

From the child's point of view, Phase II and III consisted of a game in which he had to identify the "winning" plate in order to pick a prize (a trinket). The procedure, derived from D'Antonio (1970), led the child to expect no difference between Phases II and III until he actually saw the Phase III display.

To start Phase II, *E* picked up the cans covering the displays. The child saw two plates, each with a row of mice on it. The rows differed in number (three vs. two) and either length (8.8 cm vs. 5 cm) or density (2.5 cm vs. 6.2 cm between mice). *Without mentioning any of the differences between them*, the experimenter identified the three-mouse display and the two-mouse display the "winner" and "loser," respectively. The following instructions were given to the child:

> Now, what do you see on the table in front of you? [referring to the cans]. What do you think is under those cans? Take a look. Okay, now do you want me to tell you how we're going to play the game with those mice? We're going to try to find you the plate that wins you a prize. Do you like to win prizes? Good. Well, this is the winner [while pointing to the three-mouse display]. Whenever you find this under the can like this [covers, and uncovers], you get to pick a prize [*E* then covers the three-mouse display]. But this [*E* points to the two-mouse display] is the loser plate, and when you find it [covers, and uncovers], you never win a prize. Okay? [*E* then covers the two-mouse display.] Now remember, this [uncovers the three-mouse display] is the winner and this [uncovers the two-mouse display] is the loser. Are you ready to start playing the game? [The *E* covers the plates and begins to mix the cans.] First, I'm going to mix up the cans all around and around. Now, where do you think the winner is? Okay, pick it up. Is that the winner?

When the covered arrays were mixed, the child was asked to point to the can he thought had the winner, pick up that can, and indicate whether it was the winner. If the child initially uncovered the three-mouse display and correctly identified it, he received a prize, and a new trial of hiding and mixing began. If a child first uncovered the two-mouse display and correctly identified it, the display was covered and then, without mixing the cans, *E* asked the child to find the winner. Children invariably pointed to the other can, which contained the three-mouse display. If, when they uncovered it, they identified it correctly, they were given a prize. Then there was another turn at hiding and mixing. Every lifting of a can, whether preceded by mixing or not, counted as a trial—to control for the amount of exposure to the displays. Phase II continued for 10 or 11 trials (depending on whether the three-mouse display was uncovered on trial 10). On half the trials the child was encouraged to cover the displays and shuffle them so as to guarantee he saw that the same plates were being hidden each time. Identification errors were corrected.

On three randomly chosen trials the child was asked why the display he had uncovered was the winner (or loser). The *E* never told a child why a display won or lost nor reinforced a child's answers to probe questions.

Note that Phase II was an identification task, not a shell game. That is, the child's task was to identify correctly the winner or loser after each uncovering of a plate. Children were not expected to keep track of the winner while it was being covered and shuffled. Indeed, they were allowed to play with their prizes and talk to *E* in order to minimize the probability of their focusing on the winning display when *E* set up a trial. This was done to make it possible for *E* to change the display quickly as she covered it and thereby begin Phase III without drawing S's attention to her. And indeed, by the end of 10 trials, the child was so immersed in the game and busy with his prizes that he no longer observed when *E* set up the trial. This inattention, as well as the size of the cans (which blocked a child's view of *E*'s hands), made it easy for *E* to transform the three-mouse display surreptitiously and thereby initiate Phase III.

To start Phase III, *E* covertly removed one mouse from (subtraction) or moved the mice on (displacement) the three-mouse display. The transformed arrays resulting from these *unwitnessed* operations are shown in figure 1. Phase III continued until the child uncovered the transformed array. Here, as in Phase II, the question, "Is that the winner?" was asked. The children were also asked why the display was the winner or loser; whether anything had happened and, if so, what; how many mice were present now and before; where the mice had been in the original display; who could fix the game and how.

The sessions were tape recorded and *E* noted overt surprise reactions.

DESIGN

The design of the experiment involved 24 independent groups of children. Four children from each age group were run in eight display conditions. As shown in figure 1, the eight display conditions represented a three-level binary hierarchy. The first level describes the variations in Phase II (i.e., which cue was redundant to the number difference). The other levels describe the way in which the three-mouse display was altered to produce the Phase III transformations. Half the subjects witnessed the effect of a subtraction of one element from the winning array (from the end or middle of a row); the other half witnessed the effect of a displacement (by shortening or lengthening) of the elements in the winning array. The main comparison of interest here involves the variation in operations (subtraction or displacement) performed between Phase II and Phase III.

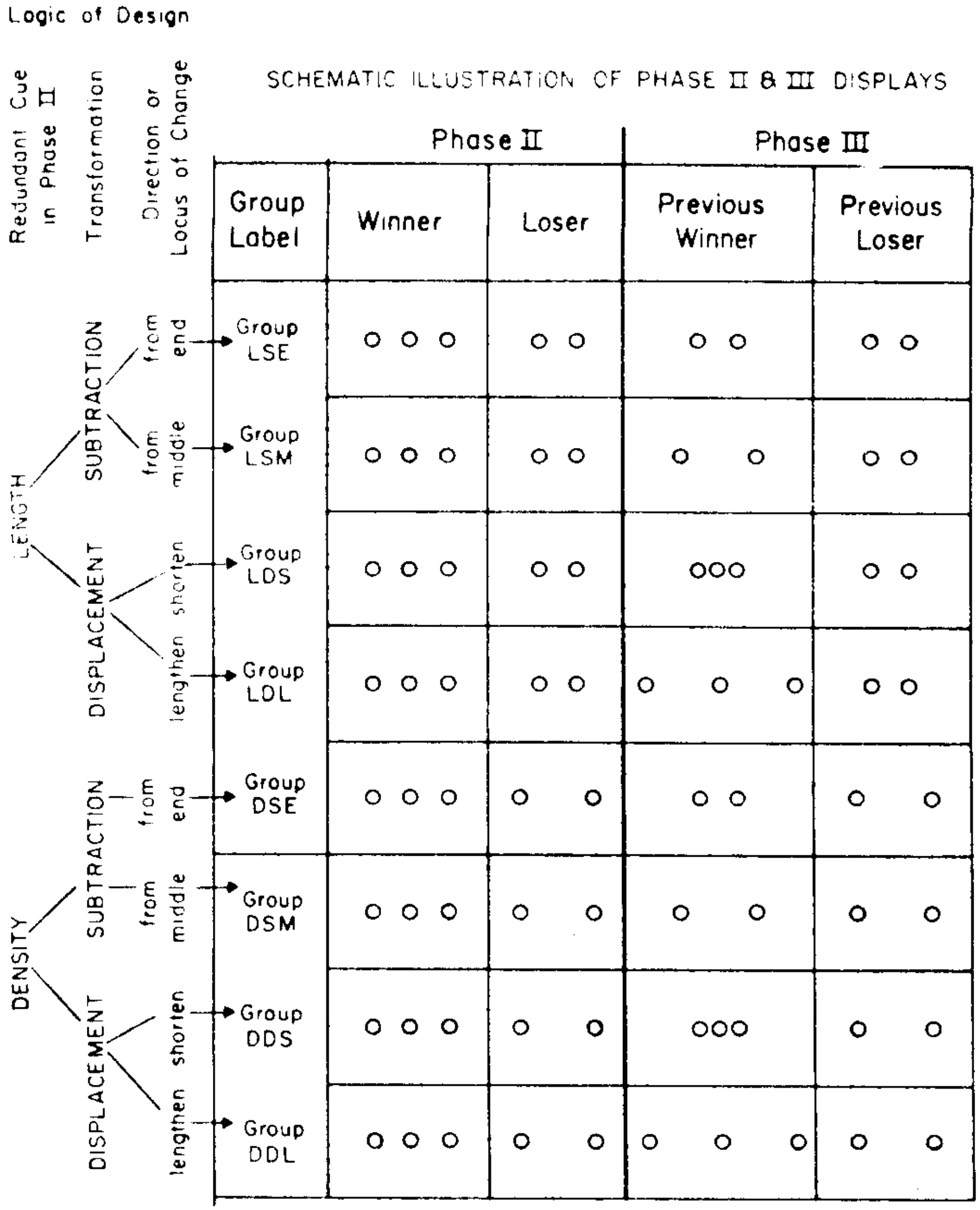

FIG. 1.—Schematic presentation of the eight Phase II–Phase III display conditions used in experiment 1.

The changes in length and density in the displacement conditions were as great as any produced by subtraction.

RESULTS

Identification Errors

All children learned to identify the winner and loser during Phase II. Eighty-three made no errors; the remainder made an average of 1.77 errors and were correct on at least five or six successive trials. More 3-year-olds made at least one error although a χ^2 analysis of the number of children who did or did not make errors in each age group failed to show a significant effect of age, $\chi^2(2) = 5.19$, $p < .05$. The nature of the redundant cue did not affect the likelihood of erring; five children in the length-redundant conditions erred in Phase II versus eight in the density-redundant conditions.

Nature of Expectancies

Two results indicate that in the course of Phase II children chose number and not length and/or density as the relevant attribute of the set. First, all but three children in each age group gave number descriptions of the displays during the identification task and/or when they saw the altered display. On all but three occasions these descriptions were accurate (e.g., "it's got three and two"). No child referred to the differences in length or density.

Second, in Phase III all but three of the 48 children in the displacement conditions continued to identify the three-mouse display as the winner even though it was now shorter or longer. The remaining three first said it did not win, then changed their minds. In contrast, changes in number in Phase III produced clear signs of doubt about winning in all but four of 48 children. Thirty-three stated that the transformed display was now a loser; the others either would not answer the question, or shifted their response from "yes" to "no" and vice versa.

In comparing the reactions produced by displacements with the reactions produced by subtractions it should be borne in mind that the changes in length or density that occurred of necessity in the subtraction conditions were no larger than the changes in length and density that occurred by design in the displacement conditions. Thus, the differing reactions imply that children spontaneously focused on number per se in forming expectancies about the arrays.

Reactions to Violations of Expectancy for Number

Because the above results indicate that children formed expectancies for number during Phase II, their reactions during Phase III can be used to assess whether they had number invariance rules. That the children said a reduced set became a loser whereas a rearranged set remained a winner suggests they did treat number as invariant. Four additional lines of evidence provide support for this hypothesis: (*a*) a comparison of surprise reactions; (*b*) the relative probabilities of the children's noticing the changes and being able to describe them; (*c*) the relative amounts of search behavior; and (*d*) the children's accounts of the nature of the intervening operations. A summary of these analyses is shown in table 1.

TABLE 1

SUMMARY OF DISPLACEMENT AND SUBTRACTION CHILDREN'S REACTIONS IN PHASE III FOR EACH AGE GROUP IN EXPERIMENT 1

	PHASE III CRITERIA				
CONDITION AND AGE	Mean Surprise Score	Noticers (*N*)	Searchers (*N*)	Noticers Who Explain Adequately (*N*)	Noticers Scored Reversers (*N*)
Subtraction:					
3 years	1.44	16	16	14	12
4 years	1.2	16	13	16	12
5 years	1.75	16	13	15	13
Displacement:					
3 years	0.25	10	0	8	5
4 years	0.50	8	0	8	8
5 years	0.50	7	0	5	5

Surprise scores.—Surprise reactions were rated as 0 (no discernible surprise), 1 (minimal surprise), and 2 (moderate to extreme surprise). A hesitation, pause, or slight voice inflection or depression was rated as 1; multiple hesitations and pauses, a child's pounding of his head, screams and exclamations, or combinations of these as 2. The agreement between two independent ratings was 95%.[1]

Of the 48 children in the subtraction condition, 28, 13, and 7 received surprise rating of 2, 1, and 0, respectively; of the 48 in the displacement condition, 3, 10, and 35 received surprise ratings of 2, 1, and 0, respectively. Children were surprised by number changes; they showed little or no surprise to length or density changes. Chi-square comparisons of age and surprise rating failed to show an effect of age in either of the two conditions.

Noticing.—If a child showed surprise, changed his response to the standard question, or mentioned some aspect of the change in the array, he was judged to have noticed the change in Phase III. All 48 children in the subtraction conditions noticed a change, as opposed to only 25 of 48 children in the displacement conditions. Table 2 presents the number of children in each age group and displacement condition who noticed a change. It can be seen that there is no age effect here and that most of the displacement children who did notice a change (20/25) were in the displacement conditions that involved moving the mice closer together.

Forty-eight percent of the displacement children who did notice a change failed to be surprised by it. By contrast only 15% of the children in

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[1] Disagreements here and in other ratings were resolved through discussion.

TABLE 2

No. Children in Displacement Conditions Who Noticed Change between Phase II and III in Experiment 1

Age (Yrs)	LDS	LDL	DDS	DDL
3	4	1	4	1
4	3	1	3	1
5–6	2	1	4	0

the subtraction condition failed to be surprised. Again, these data imply that the children who noticed displacements did not think length and density were relevant dimensions for determining which array was the winner.

Search.—A child received a positive search score if, after he saw the transformed array, he looked around the room, under the table, under the cans, etc., or asked search-type questions (e.g., "Where is it?" "Where did it go?"). Agreement between two independent ratings of search was 100%.

Children in the displacement conditions never searched. In contrast, 86% of the subtraction children showed search behavior.

Explanations.—Children's verbalizations and concomitant behaviors were analyzed to determine: (*a*) why they thought the Phase III display was a loser or winner; (*b*) if they could adequately explain what happened; and (*c*) if they knew how to reverse the change.

Children's explanations of observed changes were rated as adequate, ambiguous, or inadequate. If a child in a subtraction condition talked about a missing mouse, said one was taken away, or asked where the other mouse was, the verbalization was rated as adequate. Such verbalizations revealed a belief that there had been a subtraction of one element. It was also possible for a child to think a substitution of a two-mouse array for the three-mouse array might have occurred. Since this was a logically defensible account of the events, substitution explanations were also rated as adequate. Subtraction children who explained the change in terms of displacement only (e.g., "They moved apart") were rated as inadequate. Otherwise verbalizations were rated as ambiguous. For displacement conditions, verbalizations and/or supporting behavior that postulated movement, for example, "They moved out like this" (while moving hands apart), or a substitution were scored as adequate; those postulating subtraction or addition, as inadequate. Other accounts were scored as ambiguous. (Agreement between two independent ratings here was 93% and 98% for subtraction and displacement protocols, respectively.)

To be counted as a "reverser," subtraction children had to say they needed another mouse, or ask for another mouse to add to the display, or either take one mouse off one plate and add it to the other, or say they could or wanted to do the latter. Displacement children had to talk of the reverse displacement and/or try to perform it themselves. Agreement between two independent ratings was 99%.

When children in the subtraction conditions were asked why they were doubtful about identifying a winner, 90% said the plate now had two. When asked how many there were before, 80% said three. The ratings of children's explanations of what happened in the subtraction conditions revealed that 42 children provided adequate explanations. No child gave an inadequate explanation. Of the 42 children who gave adequate explanations, 34 indicated that there had been a subtraction of one mouse; eight thought a substitution had occurred. An example of a subtraction explanation was J. E. (3-10), who said, "There was three animals. In the can [looks around]. Took one. 'Cuz there's two now." An example of a substitution explanation was given by B. B. (5-0), who said, "Now where d'ya put the three's? You got the three's . . . . If the mouse was off [removed] that would be broken! Or ripped [pointing to a two-mouse plate]. You took the plate." Thus, children in the subtraction conditions knew the number had changed, how it could have changed, and decided this change meant they probably could not win any more because of the difference in number.

There were 25 children in the displacement conditions who were judged to notice the change. Of these, three said nothing had happened when questioned; 21 gave adequate explanations, all of which appealed to displacement. Only one child said the number changed. A typical example of explanations in these conditions come from S. W. (3-6), who said, "These pushed together. When you turned 'em in the can." When these children were asked why they won, 20 said because the plate had three on it. Thus, even though these children noticed the changes in length or density and adequately accounted for them, they behaved as if these changes were irrelevant. They still won because there were three mice on the plate.

Together, the reactions to Phase III indicate the children treated subtraction as an operation that was relevant and displacement as one that was irrelevant to number. This indicates they had the ability to treat number (at least small ones) as invariant. Lending support to this general conclusion is the evidence that the children's invariance schemes contain rules for reversing operations. Seventy-seven percent of the children who noticed the transformations behaved as if they knew that addition reversed subtraction or that elongation reversed shortening (or vice versa). Thus, for example, J. B. (4-1), a subtraction subject, said he "fixed" the game by taking one mouse off one plate and putting it on the other; and L. S. (3-8), a displacement subject, said of mice moved together, "I would spread them out."

EXPERIMENT 2: ADDITION VERSUS DISPLACEMENT

The above experiment provides indirect evidence that children know addition is an operation that is relevant to number. Many children were able to indicate that addition reverses subtraction. To obtain direct evidence on this issue the above experiment was repeated with two major modifications: (*a*) the two-mouse plate was designated the winner in Phase II; and (*b*) the subtraction conditions were replaced by addition conditions (i.e., one mouse was surreptitiously added to the winner [two-mouse] plate).

SUBJECTS

The *S*s were 35 children (17 boys, 18 girls) from two nursery schools in the Philadelphia area. In general, the sample was somewhat more heterogeneous in socioeconomic level than that used in experiment 1. Three children were dropped from the study for failure to pay attention, leaving 16 children in two age groups, with medians of 3–5 and 4–6 years.

DESIGN

The general procedure and design was as described in experiment 1. The design counterbalanced for the nature of the redundant cue in Phase II, the locus of addition (middle or end of row) or nature of displacement (lengthening or shortening) introduced surreptitiously in Phase III. The major comparison of interest here is the effect of addition versus displacement introduced after Phase II. One of the addition conditions required that larger plates and cans be used in this experiment.

RESULTS

The results for Phases II and III are very similar to those reported in the subtraction experiment.

All children solved the identification task in Phase II and most (23/32) made no errors. The remaining Ss (all 3-year-olds) made an average of 1.78 errors. There was no effect of the redundant cue variable; five children erred in the length redundant versus four in the density redundant conditions. As in experiment 1, the identification task was solved on the basis of number and not length or density. All but two 3-year-olds used number descriptions of the arrays. When faced with the altered array in Phase III, all addition Ss said it no longer won (13/16) or expressed doubt about it being the winner. In contrast, all displacement Ss said they won, even though the winning array was longer or shorter.

Reactions to the surreptitious changes introduced in Phase III indicate that Ss treated addition as relevant and displacement as irrelevant to num-

TABLE 3

SUMMARY OF DISPLACEMENT AND ADDITION CHILDREN'S REACTIONS IN PHASE III IN EXPERIMENT 2

CONDITION	PHASE III CRITERIA				
	Mean Surprise Score	Noticers (N)	Searchers (N)	Noticers Who Explain Adequately (N)	Noticers Scored Reversers (N)
Addition	1.5	16	7	10	13
Displacement	0.31	6	0	5	6

ber. A summary of Phase III reactions are presented in table 3 in terms of the main effect of addition versus subtraction without regard for age or other variables, since χ^2 or Fisher exact probability tests failed to reveal effects of these for the summarized measures. Verbal reports were scored as above, with appropriate modification of criteria for addition Ss who had to talk of the addition to receive an adequate rating for their account of what had happened and then subtraction to be counted as a reverser. Agreement on two independent ratings of surprise, adequate explanations, and whether children were reversers was at least 95%. It can be seen in table 3 that addition Ss were surprised by the surreptitious addition, and displacement Ss were not surprised by changes in length or density. The means of 1.5 and 0.3 reflect the fact that 15 addition and only four displacement Ss were surprised by the changes. All addition Ss noted the change as opposed to only six displacement Ss. Children were able to adequately explain the nature of the operation that intervened, postulating an addition or displacement depending on the condition. And, the children were very likely to indicate how a change that they noticed could be undone.

A comparison of tables 1 and 3 reveals that addition Ss were less likely to search than were subtraction Ss. This reflects the fact that the same criteria for search were used in both experiments (i.e., looking under cans, on the floor, around the room, or asking search-type questions). Since a mouse was added and not subtracted, we would not expect children to look on the floors or around the room. And indeed, no child did. Instead, search behavior was restricted to looking inside cans or under the plates as if to check for hidden mice. For example, J. S. (4-8) initially looked at the other can and when questioned said, "Something might pop out of these little bumps" (points to ridges inside the top of the can).

CONTROL EXPERIMENT

The experimental paradigm employed above yields clear evidence that, for small numbers, children as young as 3 years old possess a concept of number that is independent of the irrelevant dimensions of length and density. Furthermore, they possess a logic that treats the cardinal number of a set as invariant under spatial displacement of its elements. The logic requires that subtraction or addition operations intervene if the cardinal number of a set decreases or increases and appears to recognize that addition operations reverse subtraction operations and vice versa. These conclusions are at variance with the conclusions drawn from numerous experiments employing the Piagetian conservation paradigm. However, our experiment used small numbers (three, two) in order to minimize number estimation difficulties; whereas the conservation experiments have typically used larger numbers. It is possible that the differences in the conclusions derive solely from this variation. Piaget (1952) has argued that small numbers are processed "intuitively." By this he appears to mean that they are processed outside the domain of cognition and logic, in, say, the perceptual domain. This argument seems to imply that young children might give correct judgments in a classic conservation test with small numbers.

To assess this hypothesis we ran 10 3-year-old and 10 4-year-old children on the traditional conservation of number paradigm. The children were presented with two rows of three red poker chips in one to one correspondence. The test and assessment of conservation followed the same procedure described in Gelman (1969a). Each child has two conservation trials, one involving shortening, the other lengthening. The order of trials and the location of the transformed row were randomized.

One 3-year-old refused to answer questions, the rest of this age group judged the numbers to be different on 95% of the transformation trials. The 4-year-olds judged the numbers as different on 55% of the transformation trials. Thus, there were a total of 10 trials, one from a 3-year-old and nine from 4-year-olds, where numbers were judged equal after a transformation. Eight of the 10 judgments were backed by the explanation that both arrays had three. Only two judgments were supported by adequate Piagetian explanations (i.e., explanations that invoked identity, reversibility, compensation, and addition and/or subtraction).

Since the majority of 3- and 4-year-old children gave wrong judgments on the number conservation task and since correct judgments were only backed by logical explanations in two cases, one can conclude that 3- and 4-year-olds do not conserve number in the traditional test even when the number is as small as three. If failure to conserve number were taken as evidence for a lack of number invariance rules, then one would have to conclude that 3- and 4-year-olds lack number invariance rules even for small numbers.

DISCUSSION

The magic paradigm used in our initial experiment generates abundant evidence that very young children possess an adequate concept of basic cardinal numerosity and a logic that treats number as invariant under irrelevant transformations. When the numbers are small, as they were here, children spontaneously focus on the numerosity of a set. As a consequence, they are more surprised by changes in numerosity than by changes in length and density. Finally, and most important, in explaining changes in numerosity they appeal to addition and subtraction operations. These operations are not invoked to explain displacements. Conversely, the operations invoked to explain displacements are not invoked to explain changes in number. Of methodological interest is that the magic paradigm as such elicits revealing explanations from young children (cf. Shantz & Watson 1970).

The conclusions derived from this paradigm contradict the conclusions derived from the conservation paradigm. It is difficult to find an alternative explanation for children treating displacement as irrelevant, for correctly invoking subtraction or addition operations in order to explain decreases or increases in number, and for the children's suggesting appropriate reversing operations in these cases. The magic paradigm elicits behavior like this while the conservation paradigm does not. This leads one to ask why 3- and 4-year-olds, who possess number invariance rules, fail to "conserve." Mehler and Bever (1967) have suggested that the logical capacity of 3- and 4-year-olds to deal with number is masked by dominating perceptual strategies. However, this is not supported by our findings. It seems to us that the answer to this question lies in an analysis of the conservation task.

It is implicitly assumed that the conservation task tests for one thing—a child's logical capacity to treat quantity as invariant. Thus, if he passes the test, he is judged to possess this capacity. If he fails, he is judged to lack this capacity. On the basis of the results given here and other studies in the literature, it begins to appear that the conservation test evaluates more than one thing. Put another way, it seems that children who pass the conservation test are demonstrating many extralogical skills as well as their logical capacity; whereas children who fail are doing so for any of a number of reasons. We have argued that the problem of estimation facing a young child may lead to failure. That this is not the only problem is shown by the results of the control experiment. Here we knew estimation was not a problem since children of the same age spontaneously used the correct numbers in the magic experiment. Yet they failed to conserve. It is likely that shifts in attention can also confound the test (Bruner et al. 1966; Gelman, 1969a). When initially presented with equal rows, children may well respond to the experimenter's question on the basis of number. Then, when they observe the experimenter lengthening or shortening the row, they may redirect their attention. Closely related to the attentional problem is the problem of how the child maps language onto his logic. It may be that part of the child's problem is that he has yet to see how the words "less," "more," and "same" relate to his knowledge about numbers as well as his knowledge about other dimensions of a set and different types of sets.

Thus, it seems that the conservation task is, at a minimum, a test for logical capacity, the control of attention, correct semantics, and estimation skills. Hence, the ability to conserve represents a sophisticated level of cognitive development in which many separate abilities are coordinated. It may be that the coordination of the logic and other skills comes about with the onset of concrete operations. In any event, the child clearly possesses a logical system for manipulating number before he reaches the stage of concrete operations. It remains to determine how this capacity leads to the development of complex number skills—and how it is used with larger numbers. The results presented here are consistent with the view that simple invariance rules and a basic concept of number provide the substrate for the further development of an understanding of number properties and complex number skills.

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