Numerical Reasoning in Young Children: The Ordering Principle

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BULLOCK, MERRY, and GELMAN, ROCHEL. Numerical Reasoning in Young Children: The Ordering Principle. CHILD DEVELOPMENT, 1977, 48, 427–434. 2 experiments report on the ability of young children to reason about the numerical relations greater than (>) and less than (<). In both experiments, children were required to make inferences about set sizes of 3 and 4, based on prior training with set sizes of 1 and 2. Together the studies produced evidence that children as young as 2½ years of age could make number-based relational judgments and compare 2 number pairs on the basis of a common ordering relation. 2½-year-olds needed to be tested under special conditions before they revealed an ability to relate information obtained in 1 versus 2 training when confronted with 3 versus 4 displays.

There is considerable evidence that preschool children are able to identify the number of items in arrays of one to four and sometimes five items (Beckmann 1924; Descoudres 1921; Gelman 1972a; Gelman & Tucker 1975; Lawson, Baron, & Siegel 1974; Schaeffer, Eggleston, & Scott 1974; Smither, Smiley, & Rees 1974). Gelman (1972a, 1972b; Gelman & Tucker 1975) has shown that young children can apply some basic arithmetic principles in tasks involving small sets. Addition and subtraction are treated as operations that increase and decrease the actual number of items in an array; item substitution, color change, lengthening, and shortening are all treated as transformations that leave the number of items in an array unaltered. The research reported here addresses the question of whether the young child can use yet another arithmetic principle in dealing with small set sizes—the order relation between numbers.

Our empirical aim can be illustrated with a hypothetical experiment: an adult is asked to consider the numbers 1 and 2. He is then asked to consider the numbers 3 and 4. Finally, he is asked to compare both number pairs, indicating in which ways they are alike. He answers that in each pair one member is greater than the other, that 1 and 3 are both

less than their respective mates of 2 and 4, and that 2 and 4 are both more than their respective mates. By so answering, he reveals his knowledge of the ordering relations (<, >) that hold between two different numbers. Further, he reveals an ability to use this knowledge as a means of relating one pair of absolute number values to another. Here we ask if preschoolers do likewise.

To obtain evidence on this question, we need a task in which young children are likely to form numerical representations of two different pairs of set sizes and then compare these pairs of representations. A consideration of Gelman's (1972a, 1972b) results suggested that these criterial conditions could be met by working with a modified version of her "magic" test of numerical reasoning abilities in young children.

Gelman's procedure involves two phases. In the first phase, children are shown two arrays representing different numbers, for example, 2 and 3. The child is told one is "the winner," the other "the loser," and is then required to play a game that involves finding and identifying the winner. Although the experimenter refrains from referring to number, Gelman finds that most of her subjects use a numerical criterion to describe the winning

This research was supported by PHS grant no. HD-04598 to R. Gelman. We thank the children, parents, teachers, and administrative staff who participated in the study; and J. Baron, C. R. Gallistel, D. Osherson, and M. D. Starr for their critical reading of an earlier version of this manuscript. Authors are listed in alphabetical order. Send requests for reprints to R. Gelman, 3813–15 Walnut Street, Department of Psychology, University of Pennsylvania, Philadelphia, Pennsylvania 19174.

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and losing plates, for example, "Two loses, three wins" (Gelman 1972a, 1972b; Gelman & Tucker 1975). Thus during Phase I children abstract the two numerosities represented in the displays. In Phase II, children encounter the effects of a change in the arrays, the experimenter having surreptitiously changed the set size or some nonnumerical property (e.g., length, color) of the original set. Gelman reports that the children not only note the introduced changes but also relate what they see in Phase II to what they had seen in Phase I. A child who during Phase I encountered a twoitem loser and a three-item winner but, instead, encounters two two-item arrays in Phase II does more than state that both Phase II arrays have two items. He typically concludes that neither Phase II array wins and -as indexed by his verbalizations-infers that one item was subtracted from the original three-item array. The inference of subtraction indicates that the setting of the magic experiment succeeds in inducing the children to interrelate Phase I and Phase II observations.

We are most interested in the results from those conditions in Gelman's studies in which children encountered a change in number in Phase II. In these conditions, we can characterize the children as having formed two pairs of numerical representations, for example, 2 versus 3 in Phase I and 2 versus 2 in Phase II. Further, the children clearly related these representations. On the basis of these results, we reasoned that, if we followed Gelman's procedure but presented one versus two items in Phase I and three versus four in Phase II, our subjects should also form two pairs of numerical representations and be inclined to relate them.

On the assumption that children treat the modified magic experiment as expected, Phase II results will serve our empirical aim. Children who first learn that the one-item array wins and who are able to use a common ordering relation to compare representations of different pairs of numbers should pick the three-item array as the winner in Phase II; similarly, those initially reinforced for the two-item array should pick the four-item array. If the children lack the ability to use a common ordering relation, they should respond randomly when asked to pick between the three-item and four-item arrays. Of course the latter outcome would be ambiguous: it could occur because we erred in our assumption that the magic task would induce children to interrelate numerical representations.

There is some evidence that young children are sensitive to the ordinal properties of number, that is, to the ordering relation that holds between any two unequal numerosities. Siegel (1974) reports that children as young as 3 are able to learn to choose the more or less numerous of two arrays when the arrays represent the number combinations of 2 through 9. Should preschoolers form number-based representations of the various arrays used in our experiment and succeed in Phase II of our experiment, we would confirm Siegel's findings. In addition, we would have evidence of their ability to respond to ordinal properties of number without having been trained to do so (as were Siegel's subjects). Further, if our subjects in fact treat the Phase I arrays in terms of their absolute numerosities and nevertheless pass Phase II, that is, make a choice based on a numerical ordering relation that is common to both phases, we would have evidence of their ability to use a numerical ordering relation in an inferential way.

Experiment I

Design issues.—As indicated, children were initially shown one- and two-item arrays and subsequently shown three- and four-item arrays. The choice of set sizes and the order of their presentation were dictated by our desire to work with very young children: the smaller the number of items in an array, the greater the likelihood that young children will represent them in terms of numerosity (Gelman 1972b). Further, even 3-year-olds have some trouble discriminating set sizes of three and four if they are not first given an easier discrimination (Gelman 1972a). We selected the Phase II stimuli so that they represented numerosities not shown in Phase I and yet contained the smallest possible number of items consistent with this constraint; therefore three- and four-item arrays were chosen for Phase II. In using two new set sizes for Phase II, we prevented children from simply selecting a set size they had already encountered (as they might in a $1~{
m vs.}~3~{
m or}~2~{
m vs.}~3$ transfer condition). Given that we control for the use of such cues as density and length and show the children using numerical criteria (see below), then relationally consistent responses in Phase II would be unambiguous evidence of the young child's ability to make inferences based on numerical relations of 'greater than" and "less than" (> and <).

Subjects.—Sixty-three children (36 boys

and 27 girls) aged 2-5 to 4-11 participated in the experiment. Three of the younger children failed to complete Phase I and were dropped. Thus 24 4-year-olds (median age 4-3.5), 24 3-year-olds (median age 3-6), and 12 2-year-olds (median age 2-9) completed the experiment. Almost all the subjects attended a YWCA preschool day-care center which serves a mixed racial and SES population; five 2-year-olds and one 3-year-old were in a babysitting group in the University of Pennsylvania area.

Method.—Prior to the experimental sessions, the experimenter interacted with the children in a group setting and individually. The experiment took place in a quiet area separate from the classrooms.

The experimental setting consisted of a table on which were placed two identical tin cake pans (9 cm high × 24.5 cm in diameter). Hidden under each can was a paper plate (23 cm diameter) covered with one surface of Velcro. Small (approximately 3 cm high × 1.5 cm wide) toy animals (rabbits or ducks) were put on the plates and secured in place with a piece of Velcro attached to their base. One plate displayed one animal, the other two. All stimulus items for a particular experimental session were identical. The two-item array formed a horizontal display, varying between 3.5 cm and 6.5 cm in length for different subjects.

The experiment consisted of two phases. Depending upon which condition (more, less) the subject was assigned to, the experimenter designated either the two-item or one-item array as the winner and the remaining array as the loser. Half the children in each age group were assigned randomly to each of the two conditions. At this point, the subject saw both arrays simultaneously. During the remainder of Phase I, he saw only one array uncovered at a time. Phase I was an identification task in which subjects had to find the plate that the experimenter designated the winner. The plates were hidden under the cans, and each time the subject found and identified the winner, he received verbal verification and a plastic trinket reward (the 2-year-olds' reward involved playing with a doll). If a child uncovered and correctly identified the loser, he was allowed to uncover the winner immediately. Identification errors received verbal feedback ("No, that's not the winner. Let's mix the cans again"). Each uncovering of a can constituted a trial. Phase I lasted for 10-11 trials or continued until a criterion of five correct responses out of six was reached. Three or four times during this phase, a subject was asked why a certain array won or lost. It should be noted that the experimenter never used number terms during the experiment, nor did she use relational terms such as "more" or "less."

To begin the second phase, the experimenter surreptitiously added one animal to the two-item plate and three animals to the one-item plate. The arrays then held, respectively, three and four items arranged linearly. The length and/or density cues that differentiated the Phase II arrays varied across subjects. This was guaranteed by the fact that the experimenter had to add several objects to the arrays quickly and not be seen. As a result, the length and density of the three- and four-item rows varied haphazardly from subject to subject.

From a subject's point of view, there was no difference between the beginning of Phase II and the end of the preceding phase. The size of the tin cake covers and a subject's preoccupation with his prizes ensured that the alterations went unnoticed until the plates were uncovered in the next trial. When a child had uncovered one of the altered arrays in Phase II, he typically was surprised. The other array was then uncovered, and he was asked which of the two arrays was the winner. If a subject said that he did not know, the choice question was rephrased in an auxiliary form: he was asked which array he thought was the winner or which was best to be the winner. A child's choice was not reinforced in this phase. After the choice trial, each child was asked a series of questions designed to assess his understanding of the transformations. These included questions about what had happened, how it had happened, how many items had been on the plates earlier, how many were there now, and whether the child could fix the game, that is, change the Phase II arrays into the Phase I arrays. Finally, if possible, children were questioned about the winner in other set-size combinations, for example, 0 versus 1 and 2 versus 3.

The sessions were tape-recorded for later transcription, and the experimenter made notes on the child's surprise reactions at the start of Phase II.

Results.—In general, we found no effect of the original more-less reinforcement condition. The one analysis that did reveal an effect

of this variable is reported separately by condition; otherwise, we report the results without regard to condition.

Like Gelman, we found that the magic task tended to elicit spontaneous talk about number. It was observed that 83%, 79%, and 100% of the 2-, 3-, and 4-year-olds used number words during Phase I and/or when they first encountered the altered arrays in Phase II. Thus the children gave some evidence of behaving as if they were participating in a number task.

Phase 1.—The initial identification task was easy. No errors were made by 77% of the subjects. All but two of the subjects who did err made between one and three errors; one 2-year-old and one 3-year-old made 10 and 13 errors, respectively. A χ^2 analysis of the effect of the subjects' ages on their tendency to err was significant, $\chi^2(2) = 12.64$, p < .01, reflecting the fact that 50.0%, 33.4%, and 0.4% of the 2-, 3-, and 4-year-old children, respectively, erred at least once.

As evidenced by responses to the experimenter's probes during Phase I (e.g., "Two bunnies win; one bunny loses"), spontaneous talk at the beginning of Phase II (e.g., "Where's the one and two?"), or the Phase II question about the number of items that used to be present ("This had one" and "This had two"), a majority of the children accurately coded the numerosities of the Phase I arrays. In all, 42%, 79%, and 100% of the 2-, 3-, and 4-year-olds, respectively, met at least one of the criteria for judging a child to have accurately verbalized the numbers represented in Phase I. If a child did not refer to number on a probe trial, he typically gave irrelevant explanations: for example, "It just is [the winner]." No child referred to length or density differences.

Table 1 contains a summary of subjects' reactions during Phase II. Nearly all children noted and were surprised by the changes. Generally 3- and 4-year-olds were able to explain the changes, indicate correctly the number of items encountered, and recall accurately the number expected from Phase I.

Although most children noticed the change, it was only the 3- and 4-year-olds who made relational choices more often than expected by chance, $\chi^2(1) = 6.0$, p < .025, for 3-year-olds; $\chi^2(1) = 13.5$, p < .001, for 4-year-olds. Two-year-olds were as likely to respond nonrelationally as relationally. These conclusions are based on a child's response to either the primary question ("Which is the winner?") or an auxiliary question ("Which is the best winner?" "Which do you think wins?"), whichever the child answered. The auxiliary question was asked whenever a child failed to make a choice in response to the primary question. In fact one 2-year-old, eight 3-year-olds, and 11 4-year-olds were asked the auxiliary question. These children either made no initial response to the primary question, said they did not know which plate was the winner, or said that both plates won/lost. When asked the auxiliary question. all but one child in each age group made the correct relational response. Had we not asked the auxiliary question and scored as wrong those children who failed to make a choice response to the primary question, we would have concluded that all three age groups failed to respond relationally.

There is further evidence that the 3- and 4-year-olds were able to make an inference based on the common numerical ordering relationship represented in the Phase I and Phase II arrays. After the experimenter finished her Phase II questions, when possi-

TABLE 1 Summary of Basic Reactions to Phase II

Age Group	Mean Surprise Score ^a	% Ss Who Notice Changes ^b	% Ss Who Adequately Explain Changes ^o	% Ss Who Respond Relationally	% Ss Who Correctly Recall N Items in Phase I	% Ss Who Accurately Identify N Items during Phase II
2-year-olds ($N = 12$)	1.25	92	46	42	18	33
3-year-olds ($N = 24$)	1.63	100	74	75	63	77
4-year-olds ($N = 24$)	1.79	96	91	87	91	100

^{*} Surprise scored as 0 (no discernible surprise), 1 (some surprise), 2 (considerable surprise). After Gelman (1972b).

b As indexed by surprise, search, or verbal behaviors.

e Scored adequate if postulated the nature of the intervening transformations.

ble she showed subjects different pairs of set sizes (e.g., one vs. three, two vs. three, two vs. five) and asked them to choose the winner in these cases. Of the children asked to compare additional arrays, 85% of the 4-year-olds and 80% of the 3-year-olds made relational choices consistent with the Phase I reinforcement.

Even though 3- and 4-year-olds were able to make relational choices, few children justified their choice by using relational terms. When children were specifically asked why they made the choice they did, 28 referred to the absolute numbers represented in Phase II, while 26 gave irrelevant answers or no answers at all. Only six children gave "relational" justifications. As can be seen in table 2, only older children used relational terms in their explanations of Phase II choices: those who did typically used the unmarked form of a comparative (see Clark 1973).

The ability to respond relationally in a number task, while not dependent on the use of comparative terms, was related to a tendency to use absolute number terms correctly in both phases of the task. A χ^2 test revealed a significant effect of subjects' counting aloud and/or using the cardinal number when describing arrays in Phase I and choosing relationally in Phase II, $\chi^2(1) = 6.25$, p < .025. Similarly, children who gave relational responses in Phase II tended to count accurately and/or used number labels at some point during the second phase, $\chi^2(1) = 18.72$, p < .001.

Experiment II

The 2-year-olds in Experiment I failed to make relational responses during Phase II. One could conclude that children of this age lack the ability to make relational judgments for even small numbers. We chose the "magic" paradigm because we believed it

would maximize children's responding in terms of numerical criteria, forming pairs of numerical representations, and interrelating numerical representations. Before we conclude that the youngest children lacked the ability in question, we have to consider the possibility that the magic paradigm did not do what we expected. Although the youngest children did talk about number, thereby showing some appreciation of the fact that they were in a number task, there remains the possibility that they did not respond relationally because they failed to interrelate whatever knowledge they obtained from Phase I with what they saw in Phase II. The follow-up experiment was designed to consider this possibility.

Many investigators of cognitive capacities in young children have argued that unsuccessful performance may be due to memory constraints and not to a lack of competence for the capacity in question (e.g., Bryant & Trabasso 1971). To pass our task, it was necessary to compare the Phase II arrays with those that had been present in Phase I. Difficulty in remembering the Phase I arrays would stand in the way of deciding what was common to Phase II and Phase I arrays. Alternatively, the younger children might have been able to remember the Phase I arrays but failed to realize that information obtained in Phase I might be used to respond in Phase II. In other words, they might not have treated Phase II as a transfer task (see Trabasso, Deutsch, & Gelman 1966).

In order to test the foregoing "memory" and "transfer" hypotheses, a second group of 12 younger subjects (aged 2-5 to 3-2, median age 2-11) was tested in one of two conditions. In both conditions, Phase I was as reported for Experiment I. When the experimenter started Phase II, however, she left the Phase I arrays on the table and placed two new arrays (a three- and a four-item one) beside them. In

 ${\bf TABLE~2} \\ {\bf All~Relational~Explanations~Obtained~in~Phase~II~of~Experiment~I}$

S's Age	Set Size Reinforced in Phase I	S's Explanation for a Correct Phase II Choice
4-10	2 (more)	"Much more"
4-3	2 (more)	"Cause there's much more"
1-3	2 (more)	"It has the most—more is the winner"
4-2	2 (more)	"Four is higher than three"
3-11	2 (more)	"Cause it has all [many]"
4-6		"Because these are few, and these are many"

one condition, the Phase I arrays were left uncovered, thereby eliminating the need for subjects to recall the Phase I stimuli. In the other condition, the arrays were left covered while the child made his Phase II choice. We thought the presence of the covered Phase I arrays might serve as a clue to the demands of the task, that is, that the task required a comparison of Phase I and Phase II information. Otherwise, all features of the second experiment were as reported for Experiment I.

Five of the 12 children made no identification errors in Phase I; the remaining seven averaged 1.6 errors and made no more than 2. Five children responded to the Phase I probe trials; all of these referred to the numerosities in the array and did so accurately. Again, we find no effect of being assigned to either the more or less condition.

Of the 12 subjects, 11 chose the Phase II array which honored the relationship for which they were reinforced during Phase I. Thus there was no effect of covered or uncovered condition on the child's tendency to use a common relational response. The one effect of condition showed up in the verbal responses for Phase II: four of the children in the uncovered condition spontaneously volunteered the number of items displayed in the second pair of arrays. No child in the covered condition did so. Still, when children in both conditions were asked to indicate the number of items in both sets of arrays, nine did so accurately; the other three children appeared to be counting, that is, they at least used strings of number words. The one child who failed to respond relationally in Phase II was included in this latter group of inaccurate

The variations introduced in the followup experiment resulted in two outcomes. First, they made it easier for the children to indicate the number of items involved in the comparisons. Second, they uncovered an ability of young children to relate two pairs of arrays on the basis of a common ordering relationship. The failure to find a difference between covered and uncovered conditions with respect to the main question of interest, that is, the ability to respond to common ordering relationships on the basis of number, indicates that the young children in Experiment I did not assume that they were to compare an initial learning phase with a transfertest phase. When provided with a simple clue to this effect, younger subjects demonstrated

their capacity to use a common ordering relationship to compare pairs of set sizes. Moreover, the experimental manipulation which allowed us to observe this ability also served to make it easier for the children to reveal that they could represent the arrays with respect to their numerosities.

General Discussion

The experiments above demonstrate that under optimal conditions children as young as 2½ years can judge the ordering relation between two numerosities. Furthermore, they can use two such judgments as a basis for relating one ordered pair of numerosities to another. The conditions under which the youngest children manifested these abilities serve to highlight the care that must be taken to remove task ambiguity in experiments designed to assess cognitive capacities of very young children. Many tests of the young child's cognitive ability assess not only the capacity the experimenter seeks to uncover but also the child's ability to "read the experimenter's mind." Experiment I is an example. To pick a winner in the second phase of the experiment on the basis of the ordering relation between the two Phase II arrays, the child must of course be able to note the ordering of a number pair and use the noted relation as a basis for comparing the attributes of the Phase II number pair with the attributes of the Phase I number pair. Even if the child has this ability and in fact performs the necessary computations and comparisons in the privacy of his mind, he may fail to demonstrate this knowledge when he picks a "winner," due to his uncertainty about "the rules of the game." For all the child knows, the experimenter might have defined the winner with reference to a particular numerosity and not a relation between that numerosity and the other one. Indications that this uncertainty occurred in Experiment I are found in the fact that some children did not make a choice response until an auxiliary question encouraged them to apply whatever criterion they could think of to define a winner in the new circumstances. When children were given the auxiliary question, they invariably made a choice; and in all but three (of 21) cases they chose in accord with the relational consideration.

This sensitivity to the ambiguities inherent in the experimental protocol was most evident in the performance of the older children. The 2-year-olds, unlike the older children, did not hesitate to choose a winner in response to the primary question. In Experiment I they responded easily to the primary question in Phase II-but not on the basis of relational considerations. The 2-year-olds' random choice of a winner in Phase II might lead one to conclude that they were incapable of interrelating Phase I and Phase II stimulus pairs. Experiment II demonstrates, however, that this was a disinclination rather than an inability. The effectiveness of our effort to encourage the children in Experiment II to keep the Phase I experience in mind when making a Phase II judgment suggests that the very young child's problem is knowing when to transfer. One aspect of cognitive development between the ages of 2 and 4 appears to be an increasing tendency to reflect upon how information in one situation may transfer to an analogous situation (see Baron, in press, for a discussion of the importance of this tendency as a factor in determining individual differences in adult thought processes).

Children in Experiment II were better able to abstract the numerosities of the arrays than were children of a comparable age in Experiment I. We thus consider the limits of the magic paradigm from still another perspective. The surreptitious alteration of arrays may have introduced a general confusion factor. Children are typically surprised and get caught up in searching and asking what happened. At the same time, they are asked to indicate how many items were present, how many have appeared, etc. In the second phase of Experiment II, the surprise and confusion due to surreptitious changes are absent, and perhaps this is why the children fare better on target questions about numerosity. We were impressed that our youngest subjects in Experiment I seemed to be unclear about what was required of them at any one time, while those in Experiment II were not. Again, we see the need for special care in designing tasks to assess the cognitive abilities of very young children.

We have presented data indicating that children can make a decision based on the integration of information derived from two ordered pairs of numbers. One might interpret these results as demonstrating something akin to an ability to make transitive inferences. They do not. Transitive inference in the technical, that is, mathematical, sense can only be unequivocally demonstrated using algebraic entities. By algebraic entities we

mean symbols for numerosities other than actual number of items and, furthermore, symbols (like a and b or John's wealth and Mary's wealth) that have a purely conditional reference to specific numerosities (unlike the symbols 1 and 2). When a child is shown sets of two objects and one object, then sets of three objects and two objects, and finally sets of one and three objects, he may use the principle of transitive inference to deduce from his first two observations that the three-object set is larger than the one-object set, but he need not. He could make this judgment directly from the stimuli rather than by transitive inference. The only way to test unequivocally for the use of transitive inference is to prohibit direct perception of the direction of the numerical ordering relation between the test pairs. Suppose one is informed that John has more money than Mary and Mary more than Josephine. If one is then asked whether John or Josephine has more, the answer "John" must be derived from the relation between John's wealth and Mary's wealth and the relation between Mary's wealth and Josephine's wealth. The comparison between John's wealth and Josephine's wealth cannot be made directly—unlike the comparison between 3 and 1. For this reason, as well as for the more obvious reason that both numerosities in Phase II differed from those in Phase I, our results do not speak to the question of whether young children can make transitive inferences.

Implicit in the preceding discussion is the assumption that children who responded relationally in Phase II were comparing specific numerical representations. Evidence for this assumption comes from two main sources: the ubiquitous tendency of children to make reference to number, either on their own or in response to questions; and the significant relationship between a child's tendency to count or label correctly the number of items in sets and choose relationally in Phase II. Given such evidence, our assumption that the children represented the sets in terms of their specific numerosities is plausible. Gelman (1972b) has previously argued that the ability to represent specific numerosities is a precondition for numerical reasoning abilities in young children. The results reported here support this position. Insofar as Piaget (1952) requires children to make judgments of equivalence and nonequivalence on the basis of a one-one correspondence principle, success at his tasks can

be viewed as the development of a stage in which the child can reason numerically in the absence of representations of numerosities.¹

References

- Baron, J. Intelligence and general strategies. In G. Underwood (Ed.), Strategies in information processing. London: Academic Press, in press.
- Beckmann, H. Die Entwicklung der Zahlleistung bei 2-6, jährigen Kindern. Zeitschrift für angewandte Psychologie, 1924, 22, 1-72.
- Bryant, P. E., & Trabasso, T. Transitive inferences and memory in young children. *Nature*, 1971, 232, 456-458.
- Clark, E. What's in a word? In T. E. Moore (Ed.), Cognitive development and the acquisition of language. New York: Academic Press, 1973.
- Descoudres, A. Le Développement de l'enfant de deux à sept ans. Paris: Delachaux & Niestlé, 1921.
- Gelman, R. Logical capacity of very young children: number invariance rules. Child Development, 1972, 43, 75-90. (a)
- Gelman, R. The nature and development of early number concepts. In H. W. Reese (Ed.), Advances in child development and behavior. Vol. 7. New York: Academic Press, 1972. (b)

- Gelman, R., & Tucker, M. F. Further investigations of the young child's conception of number. Child Development, 1975, 46, 167-175.
- Lawson, G.; Baron, J.; & Siegel, L. The role of number and length cues in children's quantitative judgments. Child Development, 1974, 45, 731-736.
- Macnamara, J. A note on Piaget and number. Child Development, 1975, 46, 424-429.
- Piaget, J. The child's conception of number. New York: Norton, 1952.
- Schaeffer, B.; Eggelston, V. H.; & Scott, J. L. Number development in young children. Cognitive Development, 1974, 6, 357-379.
- Siegel, L. Development of number concepts: ordering and correspondence operations and the role of length cues. *Developmental Psychology*, 1974, 45, 693-699.
- Smither, S. J; Smiley, S. S.; & Rees, R. The use of perceptual cues for number judgment by young children. Child Development, 1974, 45, 693-699.
- Trabasso, T.; Deutsch, J. A.; & Gelman, R. Attention in discrimination learning of young children. Journal of Experimental Child Psychology, 1966, 4(1), 9-19.
- Zaslavsky, C. Africa counts. Boston: Prindle, Weber, & Schmidt, 1973.

¹Macnamara (1975) seems to suggest that the use of a one-one correspondence principle does not constitute the use of number. We are inclined to the view that a one-one correspondence procedure is not the typical procedure used to represent and compare numerosities. Still, there are clear cases in which it is. Zaslavsky (1973) illustrates its use to circumvent ritual taboos regarding the counting of people, cattle, and valuable possessions. Nontaboo items are placed in one-one correspondence with the taboo items, and then the former are counted. An inference based on the one-one correspondence of the taboo and nontaboo items yields the answer as to how many people, cattle, or valuables are present.