8 Counting in the Preschooler: What Does and Does Not Develop

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For some years now, I have contended that preschool children can and do count to represent the numerical value of a set of objects or pictures. I do not mean that the young child who is able to rattle off the number words is necessarily able to count. He may or may not be able to do so. It all depends on what else he can do with the list of words he rattles off. And as we shall see, the young child who is unable to rattle off the conventional words in the conventional order may nevertheless be able to count. In short, I do not rest my claim that young children can count on their ability to recite the conventional number words. If not this, then what? To what kind of evidence can I possibly be appealing? To answer this question, it is necessary to consider what is involved in counting. Thus, I begin my discussion with a summary of a counting model on which my husband and I have been working (Gelman & Gallistel, 1977). Next, I present data on the extent to which young children's "counts" are governed by the counting principles outlines in the model.[1] Finally, I address the questions of what does and what does not develop.

THE COUNTING MODEL

What does it mean to say that a young child counts to represent number? Inspection of the various count sequences that I have recorded led me to the view that the young child's ability to count is governed by several principles and

[1]The word *count* is in quotation marks to reflect the fact that we have yet to define the ability to count.

that adherence to some of these principles requires the coordination of several component processes. I present the counting procedure and its development by introducing the principles one by one and by examining the component processes underlying adherence to each of them. In proceeding principle by principle and component by component I make explicit the possibility that children may possess some counting principles but not others; that the individual principles could consist of several component skills, not all of which may be perfected at a given age; and that some of the principles may operate more or less in isolation in the "counting" behavior of very young children. In the end, successful counting must surely involve the coordinated application of all the principles. Indeed, as we shall see, some principles of counting require the coordinated use of other principles. Still, it is of considerable interest to determine whether any of the principles might be followed by the very young child. Should we focus solely on the appearance of completely accurate counting, we run the danger of underestimating the young child's knowledge of the counting principles. Worse yet, we might reach a conclusion of no competence, when in fact there is a partial competence. Clearly, partial competence or a limited ability to coordinate the counting principles is not the same thing as a complete lack of competence.

To account for full understanding of counting, it is necessary to appeal to five counting principles. The first three principles deal with rules of procedure or the "how to" of counting and are likely to be familiar because they are fundamental to many other counting models (e.g., Beckwith & Restle, 1966; Klahr & Wallace, 1973; Schaeffer, Eggelston, & Scott, 1974). The fourth deals with the definition of countables or the "what to" of counting. The fifth involves a composite of features of the three how-to-count and one what-to-count principles.

The Counting Principles

The One–One Principle. It would be impossible to claim that an individual kows how to count if he were unable to follow the one–one principle. The use of this principle involves the ticking off of items in an array with *distinct* ticks in such a way that *one and only one* tick is used for each item. For a child to follow this principle he has to coordinate two basic component processes: *partitioning* and *tagging*. *Partitioning* involves the step-by-step maintenance of two categories of items – those that have not yet been counted (set U in Beckwith and Restle's notation) and those that have already been counted (set C in Beckwith and Restle's notation). Items must be transferred (either physically or mentally) *one at a time* from the U set to the C set. Tagging involves the summoning up, one at a time, of distinct tags. In the successful use of the one–one principle, tagging and partitioning must proceed in lockstep. As an item is transferred from the to-be-counted to the counted category, a distinct tag must be withdrawn from the set of tags and set aside, not to be used again. In other words, tagging and partitioning must start together, stay in phase throughout their use, and stop together.

Having outlined the component processes that contribute to the one–one principle, it becomes clear that a child who fails to adhere to the one–one principle can err in at least three ways. He can err in the partitioning process; for example, he may move two items into the "counted" category on one count. He can err in the tag withdrawal process; for example, he may use the same tag twice on different items. Finally, he can fail to keep the partitioning and tagging processes coordinated; he may keep withdrawing tags when he finishes transferring items into the already-counted category.

The Stable-Ordering Principle. Counting obviously cannot proceed unless the one–one principle is put into practice. Still, there is more to counting than the ability to assign tags to the items in an array. To be credited with the ability to count, one must demonstrate the use of at least one additional principle – the stable-ordering principle. That is, there must be evidence that the tags used to correspond one-for-one to the items in an array are produced in a stable, that is, repeatable, order. This principle calls for the use of a stably ordered list that is as long as the number of items in an array. We might expect the young child to have some difficulty with it, for it presents a rather formidable serial-learning task. We might also expect to find that the extent to which young children adhere to the principle depends on set size.

The Cardinal Principle. The two preceding principles involve the selection and application of tags to the items in a set. The cardinal principle deals with the fact that the final tag used in a series of tags has a special significance. This tag, unlike the preceding ones, represents a property of the set as a whole – its cardinal number.

So, not only must one be able to assign tags in a fixed order, one must be able to pull out the last item assigned and somehow indicate that it is the item that represents the number of items in the array. To the extent that the singling out of a particular item to represent cardinal numerosity requires additional processing steps, this principle should show a delayed developmental function as compared to the one–one and stable principles. Put differently, because the application of the cardinal principle presupposes the application of the first two principles, it should be more difficult to use and therefore later to appear.

The Abstraction Principle. I have thus far focused on the "how to" of counting and have been quite evasive about the "what to" of the matter. I confess to having done this intentionally because I want to highlight the question of what constitutes countables. Those who have little intercourse with developmental theories and the assumptions they make regarding the ability of young children to count may wonder way. After all, adults behave as if they assume any collection (real or imaged) of entities can be counted. Although adults may not regard it as reasonable, they can nonetheless be induced to count sets that are made up of widely different entities, for example, a set consisting of all the great minds, all the chairs, and all the pencils in a room. This seems so obvious that one might well ask; Why elevate it to the status of a principle? There are two reasons. First, I suspect that the overgeneralization of this principle is what produces bewilderment about the claim that one cannot count all the points on a line. Second, it is an open question whether young children appreciate that the counting procedure can be applied to minds or even sets of heterogeneous objects. Indeed, many have suggested that the young child places severe restrictions on the nature of what constitutes a countable collection. For example, Ginsburg (1975) maintains that early counting – and the concept of number as well – is "tied to particular concrete contexts, geometric arrangements, activities, people, etc. [p. 60]." Gast (1957) devoted a lengthy monograph to the supposed lack of abstractness in the young child's conception of number and concluded that "Enumeration is possible for 3- and 4-year-olds only when the things to be counted are identical to one another. . . elements that vary in material composition or qualities (such as color) are not included in the enumeration [p. 66]."[2]

Work by Gast and others suggests that young children behave as if a collection of heterogeneous items does *not* constitute a collection of countables. Unlike adults, they seem to restrict severely the definition of stimuli to which they can apply the how-to-count rules. Whereas adults allow that *any* events may be classified together for purposes of counting, there is the suggestion that young children do not. The developmental questions, then, are: What is the permissible definition of "things" that can be included in an enumeration, and how does this definition change with age? If it is indeed true that the young child believes that he cannot apply the how-to-count principles to heterogeneous materials, then we would be loathe to say that he has a full appreciation of how a counting procedure can be used to represent number.

The Order–Invariance Principle. Assume for the moment that a child consistently honors the how-to-count principles and can even apply them to a relatively wide range of heterogeneous collections. Can we conclude that he has a full understanding of counting as it is involved in the representation of number? I think not. Nothing I have said so far captures the fact that the order in which a particular set of objects is tagged is in fact irrelevant. Let me be more specific.

Adults know that any count word can be assigned to any item in an array – so long as no count word is used more than once in a given count. Furthermore, adults know that the order in which items are tagged and partitioned does not

[2] Translation by C. R. Gallistel.

matter. Given a linear array of a rabbit, truck, dog, and cat arranged left to right, it is perfectly proper to count the rabbit as "one" on one trial and the cat as "one" on another trial. Likewise, it is perfectly all right to start the count with the middle object (the dog can be "one") and then skip around until all objects have been counted. We know that the result of the counting procedure will be "conserved" as objects are rearranged. Ginsburg (1975) argues that young children do not know this and cites a number of anecdotes to support his contention.

The child who appreciates the irrelevance of the order of enumeration knows a number of facts. The first is that a count item is a "thing" as opposed to a "one" or a "two." (Notice the implicit use of the abstraction principle.) Second, the verbal tags are arbitrary designates of an object and do not adhere to that object. Third, and most important, the same cardinal number results regardless of the order in which particular objects are tagged. In general, this principle captures the fact that much about counting is arbitrary.

Obviously, the doesn't-matter principle presupposes the integrated use of the other counting principles. We would expect the child who cannot apply the how-to principles in a coordinated manner to fail a test of the doesn't-matter principle. We might also expect a child who severely restricts the definition of countables to do worse on such a test than a child who is more permissive. Before examining the data that bear on these questions, though, there is one further issue we should consider.

A Caveat: You Don't Have to Use the Conventional Count Words in Order to Count

Having outlined the counting principles, I can now return to my preliminary comments. I said that a child who can rattle off the conventional count words may not be able to count. The issue is whether he uses these words in a way that honors the counting principles. At the very least, to credit him with the ability to count, we need evidence of his understanding of the one–one and stable-order principles. I also said that a child who fails to rattle off the count words may nevertheless be able to count. It is this point that I wish to dwell on.

I draw your attention to the fact that the counting model does not specify the use of conventional count words as tags. Nor does it specify that whatever words are used be conventionally ordered. This is as it should be. One does not have to use the conventional count words in a conventional order to be able to count. I suspect that the failure to recognize this contributes to the widespread belief that many African tribes cannot count. I am sure that it has led to an underestimation of the young child's ability to count. In both cases, counting ability is compared to the typical Western adult's way of counting, a way that involves the use of the traditional number–word terms and sequence. To illustrate my objection, I want to read an excerpt from Menninger's (1969) treatise *Number Words and Number Symbols.* This particular quotation appears in a passage arguing that individuals who have the number words *one* and *two* may not be able to count. Menninger demands that they have words that are equivalent to our *three, four, five*, and so on. He writes: "When a tribe of South Sea Islanders counts by twos, *urapun, okasa, okasa urapun, okasa okasa, okasa okasa urapun* (that is, 1, 2, 2'1, 2'2, 2'2'1), we distinctly feel that they have not yet taken the step from two to three. And we realize with astonishment that these people can count beyond two without being able to count to three [p. 17]." What Menninger failed to recognize is that the members of the tribe in question are using a simple concatenation rule of ones and twos for generating their tagging sequence! It is not the list that we typically employ, but is nevertheless an effective tagging sequence.

The case of a two-item concatenation rule versus a base-ten rule is but one example of why it is not necessary to use the conventional sequence when counting. In a variety of East African cultures there is a prohibition against the counting of cattle, people, and valuable possessions. Yet the same cultures are likely to place a premium on having large herds of cattle and a large number of children. It is these two factors that probably contribute to the impression that such tribes "count" with a "one-two-many" system. Consider the well-intentioned anthropologist who studies the counting abilities of such tribes. He is most likely to assume that it is best to ask individuals of the tribe to count familiar objects. And what is more familiar than children or cattle or houses in the village? Yes, they are familiar, but they also are the very items that must not be counted! The mother who is asked to count her children is confronted with two conflicting values. She must not count her children, yet she wants to tell about the fact that she has a large number of children. What to do? Answer by saying *many, a whole lot*, or something of the kind.

But how do I know that these individuals can count and that they do not treat large numbers as undifferentiated manys? Easy: They will and do count cowry shells, which are not on the taboo list of what is uncountable. Indeed, the fact that they count cowry shells allows them to get around the taboo and determine the exact number of children, cows, and so on that they have. Using one shell at a time, they touch each of the taboo items. This done, they count the cowry shells and then infer (implicity using the transitivity of one–one correspondence) the number of valuables they have.

These are but two examples of the errors one can make when assessing the ability to count. Elsewhere I take up this issue in greater detail (Gelman & Gallistel, 1977), but I trust that the point is obvious. There is no reason to require a child to use conventional count words in a conventional order. What is it that must be assumed? Intrinsic to the counting procedure is the use of unique tags to mark or tick off the items in a collection. Furthermore, the tags must be used in a fixed order. And the tags must have an arbitrary status with respect to the objects being counted; they cannot be the names of the items or properties

of the items. The set of count words meets these criteria, but so do other sets of tags. One obvious candidate is the alphabet, which is, in fact, used to count in Greek or Hebrew. But the tags need not even be verbal. They can, for example, be short-term memory bins. Recognition of the fact that counting can proceed without the use of words that are count words or, for that matter, words at all, led us (Gelman & Gallistel, 1977) to introduce some terminology. We call the traditional count words of a given language *numerlogs*. Other tags (verbal or nonverbal) are called *numerons*. Notice that this distinction allows us to hold open the possibility that animals can count. It also allows for the possibility of young children using nonconventional or idiosyncratic count–word sequences.

THE DATA FOR EVIDENCE ON ABILITY TO USE HOW-TO-COUNT PRINCIPLES

Many of the data referred to in this chapter were collected in previous studies. In this section, I summarize the methodologies used in these studies so that readers can follow the arguments without having to refer to the other sources.

The Magic Experiments: Evidence for the How-To-Count Principles

Gelman and Gallistel (1977) have analyzed two major sources of evidence for the young child's ability to use the how-to-count principles. The first source constitutes a set of transcripts available from "magic" experiments I have conducted. Although children were not asked to count, many did so spontaneously. Thus, we began our assessment of the extent to which young children honor the counting principles by analyzing a subset of available protocols. The second data source is an experiment designed to elicit counting and is summarized later in this section.

Choice of Protocols. Two series of magic experiments were selected for analysis of the counting principles. One involved the use of two- and three-item displays, the other the use of three- and five-item displays. The former transcripts were included because they were available for 2-year-olds as well as for older children. The latter transcripts were included because they involved the largest set sizes that have been used so far in magic experiments.

Participants. The transcripts from the two- and three-item magic experiments involved children aged 2, 3, and 4 years. There were 16, 32, and 32 children in each of the respective age groups. The three- and five-item magic experiments involved independent groups of 3- and 4-year-old children. There were 24 subjects in each group.

Procedure and Materials. The magic studies were initially designed to assess whether young children know that some transformations are number relevant (e.g., addition and subtraction) and that others are number irrelevant (e.g., displacement, color change, and identity change of items). The procedure involves a two-phase experiment. First, a child is shown arrays containing different numbers of objects. He is told that one is the "winner," the other the "loser." There is no mention of number. A series of trials establishes an expectancy for the numbers presented; this is revealed by the fact that answers to "why" probe trials almost always involve the child talking about the numbers. From the experimenter's viewpoint, Phase II begins with the surreptitious alteration of one or both arrays. These changes are either number relevant (addition or subtraction) or number irrelevant (e.g., displacement, color change). The data from Phase II are used to make inferences about the young child's ability to work with number-relevant and number-irrelevant transformations. The entire experimental procedure is tape recorded on an audio recorder. These tapes are then transcribed verbatim.

As indicated, we selected a set of available transcripts and analyzed all the counting sequences that appeared. In both series of magic experiments discussed here, the arrays were made up of homogeneous items (toy green mice), and these were displayed linearly.

A Videotape Experiment: Further Evidence for the How-To-Count Principles

Main Design Features. Our second source of data on the how-to-count principles comes from an experiment designed to elicit counting. The design used heterogeneous sets of 2, 3, 4, 5, 7, 9, 11, and 19 objects. It also varied the type of display arrangement (linear versus nonlinear). The variables of set size and type of display were within subject variables; age was obviously a between-subjects variable.

Participants. The experiment included children of four ages: 2, 3, 4, and 5 years. There were 19 2-year-olds to start. Two failed to understand our instructions; the data from one was lost due to an equipment failure. The *N*s in the 2-, 3-, 4-, and 5-year-old groups were 16, 21, 19, and 15, respectively.

Procedure and Materials. An attempt was made – not always successfully – to test each child on set sizes of 2, 3, 4, 5, 7, 9, 11, and 19 for six trials per set size. Three trials for each set size were linear displays; the remaining three trials were haphazardly arranged displays (except for set size 2). Linear trials preceded nonlinear trials. The displays were of chips of various colors. All sessions were videotapes for later transcription and analysis – thus the choice of *Videotape Experiment* as the tital of this section.

Those familiar with the vicissitudes of running young children will worry about the demands our design made on the subjects. We incorporated a variety of procedural features to enhance the likelihood of obtaining cooperation. First, we displayed two set sizes at a time, designating one set as the child's and the other as the experimenter's. A child was asked to count and/or indicate the number of items on "his plate" and then on the experimenter's plate. This was done to reduce the amount of time the experimenter took to set up trials. The set size pairs were 2 and 3, 4 and 5, 7 and 9, 11 and 19. They were shown in this order. Second, we used a puppet as a cohort to the experimenter since we have found that a three-way conversation among child, puppet, and experimenter heightens a young child's interest in almost any task.

Although we attempted to run all children on all trials, the child's proclivities were also taken into account. If a child wanted to keep working with a given set size, he was allowed to do so. This meant that in some cases we obtained 20 counts of a given set size! If it was clear that a child was frustrated or unwilling to continue, the experimental session was ended. Accordingly, I will report the number of children run on a given set size when summarizing individual tendencies to follow a particular principle or combination of principles.

Data Source for Abstraction Principle

We have not run experiments explicitly designed to assess the extent to which young children restrict the definition of what constitutes a countable collection of items. Still there are experiments that allow us to examine the effect of heterogeneity on a child's tendency to count and/or abstract the numerical value of an array. A comparison of results from the magic and the videotape experiments described above provides one source, because the former involved homogeneous and the latter heterogeneous sets of materials. The source of yet further comparisons will be described in Section III.D.

An Experiment on the Order-Invariance Principle

An example of the kind of behavior we sought to study concerning the young child's understanding of the order-invariance principle introduces the nature of the test we used. Assume for the moment that we have adults count an array of heterogeneous items, say, a toy-sized chair, dog, baby, flower, and car. To start, these are arranged left to right as listed. Likely as not, the adult will begin at the left or right end of the row and count the first item as "one," the next as "two," and so on until he counts the last one as "five." This done, we proceed to scramble the array and then once again arrange them in a row. This time the objects are arranged left to right as follows: flower, car, baby, dog, and chair. The adult counts, tagging the flower as "one," the car "two," and so on. Now we leave the objects in the same positions and point to the car and ask the adult to count all objects but start with the car. Our adult complies by tagging the car as "one," the baby "two," the dog "three," the chair "four," and then returning to tag the flower as "five." The demonstration continues as the adult meets our request to count while making the car be "three," then "four," and "five." He does this by skipping around the array to include all items present in each count. In so doing, he shows that he knows that each of the count words *one* to *five* can be assigned to any of the five items in the array. Furthermore, he shows that he knows that the order in which items are partitioned and tagged does not matter. What does matter is that the tags be assigned in a fixed order and that each item be assigned a distinct tag. Which item receives a given tag is completely arbitrary. Despite the reassignment of tags, the same count and cardinal number result.

The recognition that it does not matter which tag is assigned to which item is an explicit realization of the abstraction principle and an implicit realization of the other three counting principles. Accordingly, we should expect that the abiity to honor the doesn't-matter principle depends on the extent to which a child can coordinate his application of the other principles. By this logic, the development of this principle should be delayed compared to the developmental course of the other principles.

We have run a variety of tests of the doesn't-matter principle but will report on only one of them. In all cases, the central test was like the one we put our imaginary adult through. After a child counted a set of approximately five items, he was asked to modify his counting of the objects so as to have a particular object be tagged as "two," "three," "four," and so on. Some might be surprised that we could use the modified counting task with preschool subjects. To do so requires that the child be able to understand some rather vague instructions, skip around the array as he assigns tags, think of possible ways to move the items to facilitate assigning a particular tag to a particular object, and so forth. He also must remember which object is to be designated by which count word. For these reasons, we approached the initial experiments on this principle with considerable trepidation. As it turned out, there was little reason to worry. It quickly became apparent that most children understood what we were asking of them.

Participants. Twelve 3-year-olds and 15 4-year-olds participated in this order-irrelevance experiment.

Materials and Procedure. The materials used in this experiment were small trinkets that varied in color and item type (e.g., a small chair, a baby). Each child was shown a set of five objects and asked to count these five or six times. Between count trials the experimenter rearranged the trinkets, but otherwise the count trials did not differ. If it was apparent that the child could not apply the how-to-count principles consistently, the experimenter then tested the child's ability to count with a set size of four items. Children who received four-item trials were tested with the same set size during the doesn't-matter part of the experiment. Otherwise, they were tested with the five-item array.

In the doesn't-matter part of the experiment, the experimenter began by pointing to the second item in the row and asking the child to count by making that item be "one." Then he asked the child to count again, making that item be "two," or "three," and so on. This done, the experimenter pointed to the second-to-last item in the row and asked the child to make that item be "one" and then "two." Subsequent steps in the experiment will not be presented here.

THE EVIDENCE

Magic Experiments and the How-To-Count Principles

Scoring Procedure. Each count sequence was scored separately for evidence regarding the use of each of the three how-to-count principles. We identified a count sequence in the magic protocols primarily on the basis of a child having used tag items from two well-known lists – the *number words* and the *letters of the alphabet*. A child did not have to use these tags in the conventional order or, for that matter, in any order. However, he did have to use some tags. Despite our caveat regarding the use of words as evidence for counting abilities, we had no choice here. In almost all cases, a count sequence was generated spontaneously. Because the child was not asked to count, we had no other way of defining the beginning of a sequence.

A particular count sequence was scored as providing evidence for the use of the one–one principle if it had as many different verbal tags as there were items in a given array. If a child said two different number words or letters of the alphabet when shown a two-item array, he was scored as having used the one–one principle. If he said three different tags when viewing a three-item array, he was again given credit. Consistency with respect to order of tag assignment over trials did *not* serve as a criterion in this particular analysis. The one–one analysis simply addressed the question of whether children used as many different tags as there were items in the array. This done for each trial, we then considered the tendency of a child to honor the one–one principle on all his trials. If a child met the one–one criteria on at least 90% of his trials, he was judged perfect with respect to this principle. If he met the criteria on at least 60% but not more than 90% of his trials, his application of the one–one principle was judged to be shaky.

Children did have to show consistency over trials in their use of a list of tags in order to be scored positively on the stable-order criterion. The child who consistently said "two, six, ten" when he saw a three-item array on one trial and then consistently said "two, six" when he saw a two-item array was scored as being correct with respect to his ability to follow this principle. Thus, we allowed children to use idiosyncratic number lists if they used them consistently. Of course, children who chose to use the conventional sequence of number words were also given credit. And the child who used the alphabet – another stably ordered list of terms – was likewise scored as being correct. A child's use of this principle was judged to be perfect or shaky across trials in the same way as his use of the one–one principle.

We judged children's understanding of the cardinal principle by four criteria. The first involved uniquely tagging each of the display objects and repeating the last tag, for example, "one, two, three; three!", "two, six; six!" As in this examples, there were times that the repetition of the last tag was given emphasic stress. In these cases the stress on the final tag was exaggerated well beyond the bounds of the ordinary stress patterns that characterize the ends of some English sequences. Indeed, it was more like a scream or shout. We also used two indirect criteria, indirect as contrasted with the first two. A child was scored as following the cardinal principle if he shifted from assigning the correct number of tags in one sequence to stating only the last tag on a later encounter with the same set size. Again the child could do this his own way, that is, by using an idiosyncratic list. Finally, children could be scored as being correct on *this* principle if they *correctly* stated the numerical value of the set and yet seemed to not count. Why *correctly*? To do otherwise would amount to saying that any time a child says a number word, he thinks that word represents the numerical value of a given set. Maybe; but surely unlikely.

Let me anticipate a possible objection to the last cardinal criterion: that it involves a reification of my hypothesis that correct judgments of set size are almost always based on or derived from covert, if not overt, counting (Gelman & Tucker, 1975). I do not think that is a reification, and I hope to convince you with data from the varied set-size videotape experiment. But first the findings from the magic experiments.

Results. The results are really quite straightforward. The vast majority of the children – including those in the youngest group – could and did make use of all three principles. This outcome is captured in the results regarding the children's ability to use the how-to-count principles in a coordinated fashion (see Table 8.1).

In presenting the data on the coordinated use of the three principles, I make a distinction between children who made no errors at all in their application of the how-to-count principles and children whose ability to use the three principles was shaky. If we allow that both groups of children were able to use all three count principles, then we see that even the youngest subjects were able to count rather well.

The fact that so many children can be credited with having all three count principles makes it hard to ask about their relative difficulty. The numbers in the remaining cells are small, but there is a hint. If children did not follow all

TABLE 8.1
Percentage of Subjects in Each Group in Magic Experiments Showing a Given Composite Counting Profile

Observed Profiles of Available How-to-Count Principles	2 vs. 3 Experiments			3 vs. 5 Experiments		
	2-yr.-olds (N = 16)	3-yr.-olds (N = 32)	4-yr.-olds (N = 32)	2-yr.-olds (N = 0)[d]	3-yr.-olds (N = 24)	4-yr.-olds (N = 24)
All three perfect[a]	63	57	82	–	50	71
All three – at least one shaky[b]	25	34	0	–	50	21
One–one and stable	13	3	9	–	0	4
Stable and cardinal	0	3	0	–	0	0
Never refer to number[c]	0	3	9	–	0	4

[a]Includes *S*s who used cardinal number on all trials..
[b]At least 60% of the trials involved perfect coordination of the three principles.
[c]Recall that *S*s were not asked to count or use number.
[d]Not run on these set sizes.

three principles, they tended to count the number of items in the array without assigning a cardinal value to that array. That is, they behaved in accord with the one–one and stable-order principles but not the cardinal principle.

What is not reflected in the table is the typical nature of the error made by children who used all three principles but were shaky. The errors were predominantly one–one errors of two kinds: partitioning and coordination errors. The former involved the double counting or skipping of an item in the middle of an array. The latter involved a difficulty in stopping the counting procedure. A majority of the errors involved a count of one-too-few items. Tagging errors (using the same tag more than once) were almost nonexistent.

One might have the impression that since we allowed children to use idiosyncratic lists, such lists were ubiquitous. In fat, they were not. Five of the 2-year-olds used their own lists of tags (e.g., "one, thirteen, nineteen," "two, six," "A, B"). By 3 years of age, children were able to use the conventional lists in a conventional order – at least for set sizes of five or fewer objects. I do not find the latter result too surprising. After all, "Sesame Street" is watched by many a preschooler, and the program encourages children to count. What I do take as noteworthy is the fact that some 2-year-olds construct their own lists. I am particularly impressed with the occasional child who used the alphabet instead of a number word sequence. The spontaneous use of a nonnumerical sequence suggests to us that young children have available a cognitive principle in search of an appropriate list. And the alphabet *is* an appropriate list. Likewise, I think the tendency to construct an idiosyncratic list of number words suggests that the stable-order principle is guiding the learning. How else to explain the idiosyncratic but stable lists of number words? It is exceedingly unlikely that adults go around teaching the wrong list of count words. If I am right about this interpretation, we should find that when older children are asked to deal with larger set sizes, they too will invoke idiosyncratic lists. In particular, we should expect to see the use of lists that begin with the conventional sequence but shift over to idiosyncratic sequences, for example, 1–2–3–4–5–10–9.

This was one reason for the videotape study that involved larger set sizes. Another reason was my interest in whether use of the cardinal principle does indeed lag behind the use of the other principles. In addition, I wanted to determine whether increases in set size produce systematic increases in errors and if so what might be the nature of these errors. Finally, a study involving heterogeneous items would provide some information regarding young children's definition of the domain of countables.

More on the How-To-Count Principles: The Videotape Experiment

Because children did so well at counting the set sizes used in the magic experiments, we saw little point in presenting the results principle by principle. As expected, when children in the videotape experiment encountered larger set sizes they started to err. Accordingly, we present the results principle by principle before introducing the composite scores. Data from all these analyses are given first for the 3-, 4-, and 4-year-old children. Because our 2-year-old subjects almost resisted being tested on the larger set sizes, we think it best to treat their data separately.

Scoring the Videotape Data. The videotapes were transcribed as described in Gelman and Gallistel (1977). The scoring was as described above for the magic protocols, with the following differences. Recall that in the magic transcripts we had no way to identify a trial independent of the child's tendency to use number words or letters of the alphabet. In the videotape experiment, we determined the start of a trial and thus were prepared to consider whatever followed as the response. Second, children invariably pointed when they counted. Thus, we could take advantage of the pointing behavior when scoring types of one–one errors. How we did this will be taken up when I discuss error types. Finally, we found it necessary to make some minor adjustments in the criteria used for judging a child's use of the cardinal principle (see Gelman & Gallistel, 1977, Chapter 8).

The One–One Principle. The crudest index of adherence to the one–one principle is whether or not children use as many counting tags (be they unique or not) as there are items to be counted. This they generally do, as shown in Fig. 8.1.

As it turns out, the children represented in Fig. 8.1 used only number words as tags. Thus, the question becomes one of whether they tended to use as many

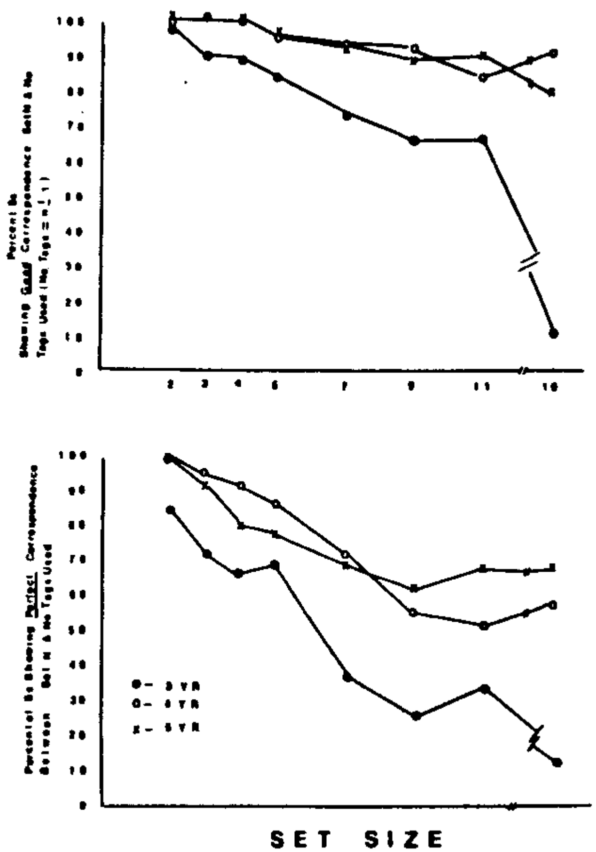

FIG. 8.1 Percentage of subjects in videotape experiment who made a good or perfect correspondence between set size and number of tags.

number words as there were tags. The bottom half of the figure summarizes the extent to which children at each of the three age levels showed perfect correspondence between set size and the number of tags they used. The top half of the figure plots the extent to which children used no more than one extra or no fewer than one less tag than the number corresponding to the set size.

The extent to which children assigned the same number of tags as there were items in the array depended on set size. Older children were better able to meet this criterion, particularly at the larger set sizes. When the 4- and 5-year-olds failed to use as many tags as items, they just missed (±1) even for set sizes of 19. The 3-year-olds did not do quite as well as judged by the N or $N \pm 1$ criteria. But still, with the exception of set size $N = 19$, their performance in the large number range is quite creditable. For set sizes 7, 9, and 11, 73%, 65%, and 67% of their trials were tagged with either N or $N \pm 1$ tags. These figures suggest

TABLE 8.2
Tendency of Children in Videotape Count Experiment to Make Each Type of One–One Error

Set Size	Age Group	Number Count Trials	N Error Trials/ 100 Trials	Rate of Each Type Error[a]		
				Tag-Duplication Errors/100 Trials	*Partitioning Errors 100 Trials*	*Coordination Errors/ 100 Trials*
2–5	3 years	402	32.6	1.0	21.6	21.9
	4 years	256	10.9	0.0	5.8	5.1
	5 years	191	18.8	1.6	11.0	6.3
7–19	3 years	122	72.1	5.7	73.0	37.7
	4 years	269	42.4	3.7	30.1	22.6
	5 years	207	35.7	1.4	23.2	19.8

[a]Error trials could have more than one error; hence the rates for the different kinds of errors do not sum to the rate of error trials.

TABLE 8.3
Rate (Occurence/100 Trials) of Each Type of Partitioning Error in Videotape Experiment

Set Size	Age Group	Number Count Trials	Type Partitioning Error			
			Double Count	*Recount*	*Omit*	*Leave Off Too Early*
2–5	3 years	402	9.7	2.8	8.9	0.2
	4 years	256	3.1	0.8	1.9	0.0
	5 years	191	2.1	0.5	8.4	0.0
7–19	3 years	122	22.6	7.3	38.0	5.1
	4 years	269	13.8	0.3	14.2	1.8
	5 years	207	7.3	1.5	12.1	2.3

that young children are quite good at applying the one–one principle for relatively large set sizes. This is also evident in the kinds of errors that children made.

Table 8.2 summarizes the number of tagging, partitioning, and coordination errors made per 100 trials for set sizes 2 to 5 and 7 to 19. As shown, tagging errors seldom occur. When errors do occur, they are either partitioning errors or coordination errors. Partitioning errors are particularly notable for those trials involving the larger set sizes. Why? Or more particularly, what does it mean to make a partitioning as opposed to a coordination error? Tables 8.3 and 8.4 present the relevant data.

Table 8.3 summarizes the nature of partitioning errors, which occur mainly when the child makes a slip in going from one item to an adjacent item. Thus, the high rate in the "double count" and "omit" columns. Children seldom leave off before finishing a count. Nor do they have much of a tendency to return to

TABLE 8.4
Rate (Occurrence/100 Trials) of Each Type of Coordination Error in Videotape Experiment

Set Size	Age Group	Number of Count Trials	Type Coordination Error			
			Beginning	*End*	*Overrun*	*Dysynchrony*
2–5	3 years	402	5.7	12.5	2.4	1.3
	4 years	256	1.2	2.7	0.8	0.4
	5 years	191	1.6	4.7	0.0	0.0
7–19	3 years	122	5.8	21.4	1.6	8.8
	4 years	269	0.0	19.0	2.6	0.9
	5 years	207	0.0	14.0	4.8	1.0

an item after it and subsequent ones have been tagged. All of this leads me to conclude that the children understand the partitioning process but are somewhat sloppy in their execution of it. The larger the set size, the easier it presumably is to lose track in the partitioning process and thus omit one item or double count an item. If this is the case, then errors should occur as the child moves between adjacent items; there should be little tendency to leave off with a number of items remaining to be tagged, nor should there be much skipping around. This is precisely what we observed.

The idea that errors in applying the one–one principle are owing to performance demands and not to a lack of competence is supported by a consideration of the type of coordination errors that occur (Table 8.4). Notice that errors are predominantly *beginning* and *end* errors. Beginning errors reflected a difficulty at initiating a coordinated application of the tagging and partitioning processes. A child would hesitate while his finger was poised over the array and then abruptly start tagging while pointing to the second item in the array rather than the first one. Or a child might drum two or three times on the first item before continuing. Similar failures in coordination occurred even more frequently at the end of an array. In all cases, errors that were classified as *beginning* or *end* errors involved a miss or double count of but one item. This is to be contrasted with *overrun* errors, which involved the continuation of tagging when there were no further items to tag or the repeated tagging of items that were displayed in a somewhat circular fashion. The dysnchrony category was used to capture those error trials on which the child's pointing got out of step with his recitation of number words or vice versa.

What to make of the fact that coordination errors were most frequently of the *end* error type and next most frequently of the *beginning* error type? I think this shows that the child has some difficulty starting the coordinated use of the processes involved in one–one, but once started he goes along just fine until he has to stop the coordinated effort. That the difficulty is so consistently focused on the last item suggests the presence of a faulty stop rule, a suggestion that brings to mind the Russian studies of young children's difficulties with tasks that require a verbal accompaniment to a motor response. The young children in Luria's (1961) study were reported to have found it hard to inhibit verbal accompaniment once they managed to start it up. Whether the comparison is warranted is not particularly pertinent to the point I wish to make, which is that the difficulty in stopping is not all that profound, for the child does stop and does so pretty much with the last item. If he misses, it is typically only by one.

With the results of the one–one error analyses, we can return to an earlier point, which is that the young child who, in the course of counting, fails to apply the one–one principle perfectly usually does so because he slips up in moving from item to item, thereby missing an item or tagging it twice. The categories of error that predominate lead to the conclusion that the child's failure indicates a lack of skill and not a lack of an appropriate underlying concept or rule.

TABLE 8.5
Number of Subjects in Videotape Experiment Who Were Not Tested at Each Set Size

	Age		
Set Size	*3 years (N=21)*	*4 years (N=19)*	*5 years (N=15)*
2	0	1[a]	1[a]
3	0	0	0
4	1	1	0
5	1	0	0
7	4	2	0
9	9	4[b]	0
11	10	2	0
19	12	2	0

[a] *E* inadvertently omitted this set size for these subjects.
[b] Includes two subjects who were not run on this set size because of experimenter's error in setting up the trial.

The Stable-Ordering Principle. Children of all three ages, when willing to count a given set, tend to honor the stable-order principle. Indeed, more than 90% of the 4- and 5-year-olds and 80% of the 3-year-olds used the same stable list on all trials, regardless of set size. Interestingly enough, many children removed themselves from the experiment when they reached a set size they could not negotiate. Thus, as shown in Table 8.5, only nine of 21 3-year-olds were willing to count set sizes of 19. We might conclude that when young children attempt to count a given set size, they apply the stable-ordering principle.

As in the previous data (from the magic protocols), we observed a tendency toward the construction of idiosyncratic lists. Unfortunately, the number of children who constructed such lists was small. There were one 5-year-old (of 15), two (of 19) 4-year-olds, and five (of 15) 3-year-olds. I say unfortunately because we had hoped to determine the relationship between set size and the tendency to construct idiosyncratic lists. For now, I simply note that when such lists appear, it is with younger children and/or larger set sizes, a result consistent with the view that the availability of the stable-order principle guides list learning. To a remarkable degree, the business of list learning is well advanced in young children, as evidenced by their facility in counting reasonably large set sizes.

The Cardinal Principle. How did the children do with regard to the cardinal principle? It is by no means the case that the children were unable to honor the cardinal principle. Three- and 4-year-olds did rather well on the smaller set sizes, and most 5-year-olds managed to arrive at the cardinal number of set sizes as large as 9 (see Fig. 8.2). Still, as compared to the one–one and stable principles,

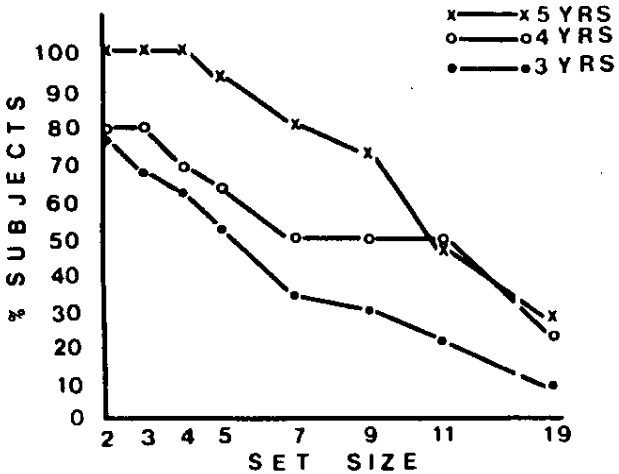

FIG. 8.2 Percentage of subjects in videotape experiment who used cardinal principle on a given set size when all criteria listed in text are applied.

on the extent to which children honor all three principles in a coordinated fashion. Before I turn to a discussion of the use of all three principles, there is an important point I want to make about the scoring criteria for the cardinal analysis.

Recall that we credit children who simply state the correct number but who do not count aloud. There was a question about the legitimacy of this criterion.

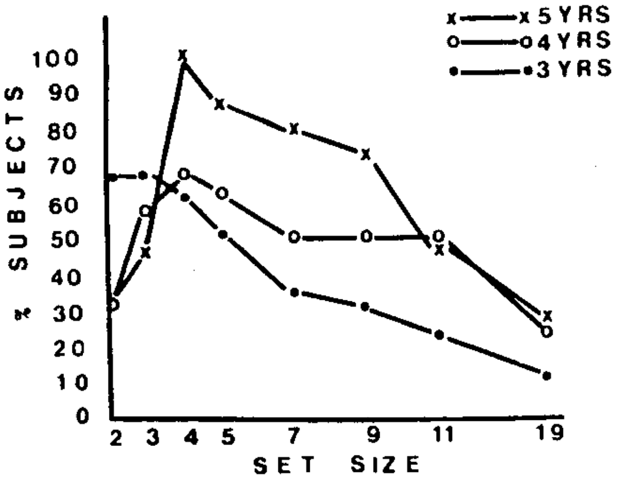

FIG. 8.3 Percentage of subjects in videotape experiment who used cardinal principle on a given set size when all criteria *except number labeling* are used.

The results of the present study allow us to address this concern. Despite the fact that the experiment involved repeated requests to count, there were children who chose not to do so – at least not aloud. As a result, we have cases where the evidence for cardinality is based on the child's having stated the correct number for that array. In Fig. 8.3 I present the results of concluding that a child who simply gave the correct answer was not using the cardinal principle – at least not in the way the counting model requires. A comparison of Fig. 8.2 and 8.3 shows no effect of shifting the scoring criteria for 3-year-olds. However, the shift in scoring criteria produces a rather odd result for the 4- and 5-year-olds; they seem to do worse than the 3-year-olds. Furthermore, they do better in applying the cardinal principle on set sizes 4, 5, 7, 9, and 11 than they do on set sizes 2 and 3. I think the only sensible way out is to conclude that some of the older children have progressed beyond the stage of having to count aloud to determine the numerical value of very small set sizes (cf. Gelman & Tucker, 1975). I also think that I am justified in assuming that a child who does not count aloud but nevertheless gives the correct answer regarding the numerousness of the set can use all three principles. This is the assumption that was followed in determining how well children coordinated the three how-to-count principles. In other words, the child who simply stated the correct numerical value of a set was assumed to have the ability to apply the one–one, stable–order, and cardinal principles for that set size. Because all children who were so scored on a small set size gave clear evidence of the coordinated use of the three principles on larger set sizes, I was satisfied that this assumption was warranted.[3] Now for the results of the analyses of the children's ability to use all three principles in concert.

The Coordinated Use of the How-To-Count Principles. When the set size is small (and presumably the demands on the child are minimal), most children apply all three principles in conjunction. Thus, for a set size of 2, 76%, 74%, and 96% of the 3-, 4-, and 5-year-olds gave evidence of using all three principles. Similarly, for set size 3 the respective percentages were 67%, 79%, and 100%; for set size 4, 57%, 68%, and 100%. By set size 7, the respective percentages fell off to 19%, 47%, and 80%; only in the 5-year-old group do we see a majority of children able to honor all three principles for set sizes 2 through 7.

Obviously, an increase in set size made it difficult for children to continue to use all three principles in conjunction. What happened? On the basis of sequential analyses we reported elsewhere (Gelman & Gallistel, 1977), increases in set

[3]The reader who is unhappy about this decision will be able to make up his own mind by considering Table 8.14 in Gelman & Gallistel (1977) wherein care is taken to present the results of altering criteria for cardinality. It is of interest, however, that when this criterion of cardinality is not followed in assessing the ability to use all three principles it again turns out that 3-year-olds are better than older children because they are more inclined to count aloud.

size first led the children to stop applying the cardinal principle. When children used two of the principles rather than all three, the two they used were the one–one and stable-order principles. It was not that the latter two continued to be applied perfectly. In fact, some errors in applying the one–one principle appeared just at the point where children stopped identifying the cardinal number of the set. As set size increased still further, so did the tendency to make one–one errors. This is reflected in the fact that we eventually found ourselves crediting children with only the stable-order principle because there were too many error trials (i.e., more than 60%) on the one–one principle.

This pattern is true regardless of age, although, not surprisingly, the younger the child, the smaller the set sizes at which he began to falter. I take the pattern to mean that at a very early age children know many of the fundamentals of enumeration. The nature of development, at least from 3 years of age onward, appears to be one of skill perfection and not the apprehension of new principles. When the set size is small and the demands on the child's skill minimal, most 3-year-olds apply all three principles in conjunction. Increases in set size produce errors in the application of the one–one principle. I think the child stops using the cardinal principle when he recognizes the uncertainty of his performance on the one–one principle – successful adherence to which is presupposed when one applies the cardinal principle.

A Return to the 2-Year-Olds in the Videotape Experiment.[4] Do 2-year-olds show any appreciation of what is involved in counting? Apparently they do. All but one of the children used lists of number words. In this regard, we identified three classes of children: One class used idiosyncratic lists, another used conventional lists, and one child used the same number word over and over again (e.g., "3–3–3"). The five (of 16) children who used idiosyncratic lists had conventional lists for the first three entries and then took up with their own lists (e.g., 1–2–3–6–5–10). One of the idiosyncratic list users recycled when he ran out of available number words and thereby produced a list of 1–2–3–1–2–3 and so on. All children who used idiosyncratic lists adhered perfectly to this order over trials. in contrast, only one of the nine children who tried to use the conventional order of numerlogs did so perfectly; the other eight met the criterion of using the same list on at least 60% of their trials and were therefore judged as being shaky in their ability to use the stable-order principle. Note that the child who follows his own list does *better* at honoring the stable-order principle than the child who follows convention.

Okay, so 2-year-olds can use a list of number words. But do they have any inkling that these words are to be assigned as tags to items in a set? I think so. First, the children pointed – to be sure, not perfectly. Still, 10 out of 16 did

[4]The number of 2-year-olds who were tested on set sizes 2, 3, 4, 5, 7, 9, 11, and 19 were 16, 15, 13, 11, 7, 7, and 0, respectively.

well enough for us to do error analyses vis-à-vis the one–one principle. And just as we found with older children, the main error types were coordination and partitioning errors involving one item. That such errors occurred led us to consider the possibility that the children were assigning approximately as many tags as there were items to tag. This brings us to another source of evidence on the one–one principle. We found an overall tendency for the 2-year-olds to use more tags as set size increased (Fig. 8.4) This is not to say that the number used was precise – it was not. Nevertheless, the number of words recited did tend to increase with set size.

Eight (or 50%) of the children were able to count and identify the cardinal number of a two-item array. The median age of the children who did not provide evidence of ability to indicate the cardinal number was 27 months; the median of children who did was 31 months. The four children who were scored as having followed the cardinal principle on a set size of 3 were at least 32 months old; indeed, three of them were 35 months old. It seems, then, that the younger the child, the less likely he is to give any evidence of following the cardinal principle – a result that is consistent with our position that a child must show skill on the one–one and stable-order principles before he can arrive at the cardinal representation.

To be sure, 2-year-olds are not skilled counters. Nevertheless, it is possible to identify the use of some components that make up the counting procedure. They attempt to tag items, they point (albeit less than systematically), and they appear to be guided by a stable-ordering principle. Yes, they have much to learn.

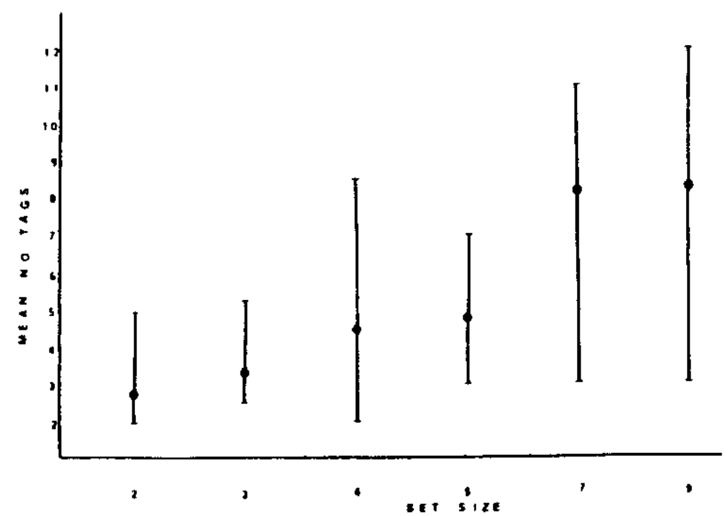

FIG. 8.4 Mean and range of mean number of tags used for each set size by 2-year-olds in videotape experiment.

But they will not be without help, for they seem to have available counting principles – principles that will guide them in their efforts to achieve performance mastery.

The Abstraction Principle

Recall that the displays in the videotape counting experiment were heterogeneous. Yet I have said nothing about the way this influenced behavior. The reason I have said nothing is that nothing seemed called for. We failed to observe any tendency for children to pick out those items that were alike (say, in color) and count only those; instead, they treated all the heterogeneous objects in our arrays as countables. We found a similar effect, or rather lack of effect, when we compared the ability of children to identify the number of heterogeneous or homogeneous items displayed on a card (Gelman & Tucker, 1975). In that study, we found that children in magic experiments take changes in identity or color of an object within an array to be irrelevant to the array's numerousness. Indeed, over and over again, I find that preschoolers are content to group together and count items of diverse composition.

Is it surprising that young children will count "things?" It is from one point of view but not from another. Developmental accounts such as Gast's (1957) are tied to a particular view of number concepts. This view holds that number concepts develop along with the child's ability to classify objects and events into organized hierarchies. It is a common (although not necessarily correct) view that children first classify together objects that share salient perceptual properties and only later develop the ability to use "abstract" criteria for the purposes of classifying. The theme that concepts are initially perceptual and then abstract (or logical) has dominated the major theoretical writings in developmental psychology. From this viewpoint it is easy to slide into the position that the ability to classify entities as "things" for the purposes of counting a heterogeneous set of objects is relatively advanced developmentally.

However, one need not assume that a complex hierarchical scheme mediates the ability to classify entities as "things." Instead, it is possible to view the ability to classify the world into things and nonthings as a derivative of the ability to separate figures from grounds. In this case, the categorization of things as opposed to nonthings may well be a very early achievement. For the child to count heterogeneous materials, he simply needs to treat them as "things." He need not know the way these things can be assigned to various levels of a classification hierarchy.

I do not mean to suggest that the young child will be willing and able to count any collection of items or nonitems I put together for a count trial. It would be amazing if there were no restrictions placed on what is countable. I know of no evidence regarding the countability of imaginary items and would not be surprised if a child said he could not count them. This is a matter for further research. Still, the young child's definition of countables seems to be relatively unrestricted.[5]

The Order-Invariance Principle

We now come to the last of the counting principles. In a sense, this principle is redundant of the other four. It captures the way the first four principles interact in contributing to a full appreciation of counting. We expect that a child who does well on the order-invariance task will use the how-to-count principles in concert. But we are getting ahead of ourselves. First, we have to consider how children performed during the doesn't-matter experiment. And to do this, we again have to consider scoring criteria.

Scoring the Data. For purposes of the present discussion, there is no need to go over all the details of the scoring procedure. It is enough to know that performance on each trial was scored at one of four levels. Then, an overall performance score, which took into account trial-by-trial performances, was assigned each child. A child could respond in one of two ways on a given trial and be scored at Level 1 – the top grade. The child who tagged the designated object as requested and made no how-to-count errors while doing so was obviously given a Level 1 score for that trial, but so were children who adopted strategies for making a given object be *X* that forced them to produce errors. For example, there was the child who, when asked to make the second object in a row be four, tagged the first object *one* and then held his finger in the air while he said, "Two-three" and then dropped his finger to touch the designated item while he said "four" and continued to count the remaining objects. There were also children who rearranged the order of their lists (e.g., one, four, two, three, five) so as to be able to proceed left to right and tag each item in succession. There was a variety of such strategies (Merkin & Gelman, 1977) that enabled children to follow our instructions but then forced them to make errors. We distinguished between strategic errors and straightforward counting errors in the application of the how-to-count principles. Trials that involved simple counting errors but otherwise followed our instructions were scored as Level 2. Still poorer performances were scored as Level 3 or Level 4.

Having reliably scored the performance on each trial at Level 1, 2, 3, or 4, we assigned each child an across-trials performance rank of I, II, III, IV, or V. Distinctions in overall performance reflect the extent to which children received Level 1 or 2 scores on their individual trials. Those children who received a Level Level 1 score on *all* trials were assigned an overall performance rank of I; child-

[5] In Gelman and Gallistel there is a discussion of the evidence that appears to contradict this conclusion. We argue that such evidence was collected in experimental situations that set children to think that they were to count together items that were alike.

ren who scored at Level 1 on at least 60% (but not 100%) of their trials were assigned an overall rank of II; children who received Level 1 scores on no more than two trials were assigned an overall performance rank of III; children who failed to receive Level 1 scores but received Level 2 scores on at least 60% of their trials were assigned an overall performance rank of IV. All other children were ranked at level V on overall performance – reflecting the fact that they had considerable trouble with the experiment.

Results. Recall that there were 12 3-year-olds and 15 4-year-olds who participated in the order-irrelevance experiment. Seventeen and 47% of the respective age groups received an overall performance rank of I. Another 25% and 20% in each age group received an overall rank of II. Thus, 42% and 67% of the 3- and 4-year-olds scored at least at rank II. The percentages of children scoring at each of the remaining ranks were 17%, 17%, and 25% in the 3-year-old group and 7%, 13%, and 13% in the 4-year-old group. These data reflect the fact that many children made some how-to-count errors when performing on the doesn't-matter items. Only 17% and 47% of the 3- and 4-year-olds were able to negotiate the doesn't-matter test items without making straightforward count errors. This raises the question of the relationship between the child's understanding of the how-to-count principles and his overall performance on the doesn't-matter items.

In this and all other experiments on the question at hand, it was the better counters who received the top grades for their overall performance on the doesn't-matter items. For a child to do well on the latter tests, he had to show independent evidence of being able to coordinate the how-to-count principles on at least some set sizes. However, an ability to apply the how-to-count principles in concert is not a sufficient ability for the order-irrelevance principle. In every experiment we have done, we find good counters who do very poorly on the doesn't-matter items. Thus, as expected, the ability to apply the order-irrelevance principle is dependent on the development of counting skill, yet counting skill is not enough. The question of what new knowledge develops and makes it possible for a child to honor the order-irrelevance principle is taken up in the next section.

WHAT DEVELOPS?

I address the question of "What develops?" by first considering what does not develop – at least from the age of about 2-1/2 years on. Even the youngest children in our how-to-count studies behaved in accord with the one–one and stable-order principles. They may have erred in the application of these principles; nevertheless, the errors that occurred are best characterized as performance errors rather than as errors that reflect complete lack of knowledge of the one–one and stable-order principles. How else to explain the tendency of children to assign approximately as many tags as there are items to tag? And how else to explain the appearance of idiosyncratic number lists or even the novel list of the alphabet? Surely, no one would expect to find an adult in this culture who intentionally and consistently taught their young to count with idiosyncratic lists of number words. And it is exceedingly difficult to imagine adults speaking to the beginning language learner as follows: "*A*, you do this; *B* you do that; *C*, you do that," and so on. It would be a violation of what speech looks like to the 2-year-olds (see Ferguson & Snow, 1977, for a review of the relevant literature). Since the use of such strings is repeatable and stable over trials, I can think of no other explanation than that the young child has a principle in search of a list.

It is perhaps a lucky thing that there is a principle to guide the acquisition of the count–word sequence. Even with its aid, the child has much to learn. The research reported in earlier sections of this chapter involved set sizes as large as 19; it did not involve set sizes of 20, 30, 100, 1000, and more. I do not know what we will find when we use such set sizes, but I venture to guess that young children will encounter considerable difficulty, for the generation of the count words involved here depends on a base–ten rule. I suspect that the child has yet to develop such knowledge. Here, then, is one candidate for development.

Another candidate for development has been highlighted in several of the preceding sections. This is the eventual perfection of skill in applying the one–one principle. For the counting procedure to work as a method for establishing the numerousness of a set, it will not do to allow partitioning or coordination errors. Over trials, a child who makes such errors will arrive at different estimates of the same set size. This will allow him to conclude that set sizes of the same number of objects represent different numbers from trial to trial. Needless to say, such a child is unlikely to conserve number. But worse yet, he is very likely to falter in making same–different judgments of those set sizes for which he still makes one–one errors. Thus, it is essential that the young child perfect his application of the one–one principle, or else his ability to compare the numerousness of sets will be exceedingly limited.

Related to the former argument as to what develops is one regarding ability to apply the cardinal principle. We have seen that the child who makes one–one errors for a given set size is disinclined to identify the cardinal number of that set. To the extent that the application of the cardinal principle depends on the coordinated use of the one–one and stable-order principles, this is as it should be. Yet it seems a bit odd to say a child can count and still not know the numerical value of the set. Until the child can state the cardinal number of a given set, it is not clear that he realizes that a given count can be used to determine cardinal number. Like others (e.g., Schaeffer et al., (1974), I conclude that the development of the cardinal principle is delayed compared to the one–one and stable-order principles.

Even when a child can apply the cardinal principle, it is not clear that he has a full appreciation of it. Recall that we observed children who could use all three how-to-count principles and still did not do well on the doesn't-matter items. A good score on the order-irrelevance tasks can be taken to reflect an understand-ing that once a given set has been counted it does not matter how it is recounted – the cardinal number therein will be conserved. The development of the cardinal number concept would thus seem to pass through stages. Initially, the child has to repeat the last tag of a count sequence. Then, he sees that the cardinal number is conserved across repeated counts. Presumably the next step is the ability to rely on a rule of one–one correspondence without having to count at all. If we assume that this is what Piaget's test of number conservation is about (cf. Brainerd, 1973), then we should expect a developmental lag between a child's ability to conserve number as revealed by the doesn't-matter test and the Piagetian test. In an ongoing experiment designed to test this hypothesis, this seems to be what happens.

In the end there should be yet one further development, the ability to reason about number without first having to count and thereby to achieve a specific numerical representation. It is only when the child is freed from his reliance on the counting procedure that he will be able to reason about algebraic entities, appreciate the rule of one–one correspondence, and so on. From Piaget (1952) we can conclude that these are the kinds of arithmetical knowledge that emerge during middle childhood.

ACKNOWLEDGMENTS

The preparation of this paper as well as the research reported in it were sup-ported by NICHD Grant No. 04598 and NSF Grant BNS-77-03327 to the author. The author's address is 3813 Walnut Street, Department of Psychology, University of Pennsylvania, Philadelphia, Pa. 19104.

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