IMPLICIT AND EXPLICIT KNOWLEDGE: AN EDUCATIONAL APPROACH

edited by

DINA TIROSH

Tel Aviv University, Israel

Human Development, Volume 6

The Tel Aviv Workshop in Human Development

Sidney Strauss, Series Editor

ABLEX PUBLISHING CORPORATION
NORWOOD, NEW JERSEY

2

Constructivism and Supporting Environments

Rochel Gelman
University of California at Los Angeles

Introduction

A major question organizes this volume, "How can educators lead learners from implicit to explicit understanding, especially in the domains of mathematics and science?" This chapter focuses on why it may be difficult to achieve this goal. For one, learners can and do find interpretations that differ from those intended by experts. Our work on fractions will be used to develop some of the implications of this point. In addition, we still do not know how to characterize relevant inputs for learning within a constructivist theory. In order to educate children about explicit theories, we need guidelines for selecting relevant inputs. Constructivist theories have yet to develop these. Our beginning efforts to fill this lacuna are woven throughout the manuscript.

The absence of a constructivist theory of supporting environments is a significant issue in its own right. But even if there were one, we still would be well advised to adopt a wait-and-see stance that is skeptical about new programs and teaching materials, for a foundational assumption of constructivist theories is that learners are actively engaged in selecting and interpreting inputs for and by themselves. Therefore, it is always possible that novices will ignore and/or misinterpret the environments we offer them, forcing us to return to the drawing board—even if we are sure that our new offerings are grand.

Even granting that we now know a great deal about the initial states of a novice's knowledge and that we should build our armament of teaching props to acknowledge these, it is still the case that our pupils could—and probably will—find interpretations that do not match the ones we intended. This follows once we grant that the mind constructs representations on the basis of what it brings to the learning setting as much as what it is offered. We should be prepared for such "failures" of attention and/or interpretation and stand ready to tune or change a program *after* it is put into the arena of learning.

Support for the reported research and preparation of the manuscript came form NSF grants BNS 8916220 and DBS-9209741. Thanks to Iris Levin for working with me on the Inside-Outside data; Betty Meck and Jason Macario for their help with the chapter; and Sid Strauss; Dina Tirosh, Tamar Zelniker, and other members of the Unit in Human Development in the School of Education, Tel Aviv University, for their insightful comments on an earlier draft of this chapter.

On the Need for a Constructivist Theory of Environments

Cognitive developmentalists (especially those influenced by Bartlett and Piag(have converged on the position that children play an active role in the acquisition their own knowledge (see Gelman & Brown, 1986, for a review). No longer do \ treat young learners as passive recipients of whatever we deem to be good for the Instead we acknowledge that even young children can find, choose, and sometim make up inputs that foster development of the representations they are bus constructing. Even when early knowledge is implicit in form, it still influences h(settings at home, in school, and the culture at large are interpreted.

Educators too talk of the importance of acknowledging the constructivist tende cies of their students. Especially in the areas of mathematics and science educatic much has been written about learners' tendencies to develop systematic and org nized knowledge bases about numbers, electricity, substance, the way physi(objects move, the animate–inanimate distinction, and so on, often before enteri school and often without any obvious guidance from others (see Carey, 1986, one review). There is a growing realization that instructional efforts have to ta into account these active tendencies of learners to make sense of inputs in terms what novices (as opposed to experts) take for granted (Glaser, 1987). Everywh(there are efforts to characterize the nature of the representations that learners bri to school with them. There is even some work on the mechanisms by which su representations are acquired and could be modified (see other chapters in t volume). Still, surprisingly little attention has been paid to a topic of cent importance for educators, namely, how to characterize the nature of supporti environments once it is assumed that learners actively interpret and select inputs the basis of their knowledge bases. In the absence of such an account we have incomplete constructivist theory of learning and development.

To say we still need a constructivist theory of the environment and its use is say we need to characterize its laws of learning. For example, what laws parallel replace the association laws of frequency and contiguity? What kinds of data foundational, that is, serve as primitives during early cognitive development? D(the associationist answer that such data are sensory bits hold for a constructi\ answer to this question? If not, what alternative does? Are we committed to notion of a tabula rasa at birth? And so on.

On Relevance

As soon as we give to the mind the ability to define what is relevant, we give ove great deal of our control of selecting inputs to that very same mind. The result is t we no longer can hold the longstanding view that we, be we scientists and educators, know what counts as relevant inputs for learning about *X*, *Y*, and *Z*. T consequence is not widely appreciated. For example, wittingly or unwittingly, continue to act as if we know what the relevant data are—no matter what the ch or novice learner of any age thinks. We continue to have the idea that those data that experts take to be relevant will also be viewed as relevant by novices.

Constructivist Theories of Mind: Empiricist Theories of Learning?

If we assume that children construct representations, that they are active participants in the buildup of their own knowledge, we must acknowledge that they also have considerable control over the definition of relevant inputs. The young might even use different data than expected and therefore construct different representations than the experts. If they do, their definition of supporting environments for learning will not converge upon ours. What kinds of assumptions about learning do we need so that novices will build knowledge bases that converge with those of their elders?

Tabula rasae? Can we incorporate the widespread assumption that our young have no representations, that they do not start out sharing knowledge with their elders? If our young have no representations, let alone any in common with us, then they are free to interpret all inputs anyway they want, in any of an infinite number of ways. On straightforward grounds of probability, the odds are that they will generate interpretations of a given input that differ from ours. Similarly, in the absence of any representations to constrain initial interpretations, the odds are high that the young will vary widely amongst themselves in their interpretations of a given input. As a result, members of the next generation should differ from their elders and among themselves in how they interpret seemingly identical inputs.

Since our commonsense observation is that our own young do come to share with us a common core of knowledge, how can we capture this observation within a constructivist account of knowledge acquisition? Feldman and Gelman (1987) offer one solution, in what they call their rational-constructivist account of cognitive development. They assume that young children share some initial knowledge that is common to ours. The knowledge is assumed to be skeletal in form, to be but outlines of some domains of knowledge. These outlines serve to define the class of stimuli that are potentially relevant to further learning about the concepts of the domain. To the extent that they do this, they also serve to direct children's attention to inputs that will nurture the domain's development, that will lead children to seek and respond to those inputs we consider relevant for learning more. Similarly, these structures serve as file drawers of memory about the noted inputs, making it possible for the child to collect relevant data about the body of knowledge.

By limiting the innate knowledge base to *some* skeletal principles, Feldman and Gelman make clear their commitment to the need for learning and development. They also provide an account of how the knowledge that is acquired by the young can converge on the shared knowledge base of the domain. These skeletons do more than focus attention on relevant data. They provide a core around which assimilation and accommodation can take place. Bits and pieces of noticed data are assimilated to an existing structure that serves to keep data in an organized and coherent way. As these skeletal structures are applied they nurture thems leading to their own fleshing out.

There Is More to the Nature of Environment Than Meets the Eye (Ear, etc.)

On the nature of foundational inputs. Even if we assume our young share skeletal cores of knowledge with their elders, they still may treat as relevant inputs that we take to be irrelevant, or nonexistent. One consequence is that lea sometimes treat as relevant some inputs that we take to be irrelevant or ar aware of! Work on language and concept acquisition in blind and deaf chi helps illustrate these points. It leads to the conclusion that those who know the about a domain are sometimes more "expert" on the subject of relevant inputs are those whose who have mastered that domain. When this does happen, w look at their solutions for clues on how to characterize more accurately the natu relevant inputs.

Some blind and deaf children learn what they "should" not. There is s widespread belief that the initial inputs for learning a concept, the primitive upon which the concept is built, must be in the domain of the target concept example, visual concepts require exposure to visual data, auditory skills re auditory inputs. If so, the congenitally deaf should be at risk for acquiring lang and the congenitally blind should be at risk for learning the meaning of visual te ones about seeing, looking, being blind, and the like. This way of characterizin requirements that initial inputs must meet has a long and distinguished history example, when John Locke discussed the implications for blindness for his em cist theory of concept learning, he concluded that blind children would not dev certain kinds of knowledge—this because they could not receive the requ foundational inputs, sensations generated by light.

As a reminder, Locke assumed that concepts are built up by associations tha based on those primitive sense data that fall on functioning senses. The assump that uninterpreted sense data have a privileged status is still very much with Many continue to hold that children learn the meaning of words by seeing some point to objects and actions in the context in which a novel label is uttered. assumption is that pertinent sensations from the target item, the spoken word, the setting are generated close together in time and/or space, the consequenc which is that all can be associated together. Repeated exposure to such pairing the sound sequence and other patterns of sensory data lead to the strengthenin associations between these. With the buildup of such associations, the so sequence begins to take on, or stand for, a meaning, one that represents the objec events responsible for the sensory data that generated the sense data (see Gelma Cohen, 1988, for a further discussion of this issue).

Presumably, the function of pointing in this account is to single out the patter visual input that should be associated with the sound of a given word. If so, bl children should have more trouble learning language than sighted children; they cannot receive that set of relevant sensations that have to do with pointing. In addition, they should be especially at risk when it comes to learning vocabulary terms of sight like LOOK, SEE, BLIND, SHOW. Knowledge of objects should be limited to those features that are defined haptically and/or aurally. Although it might seem reasonable to predict that the blind will fail to learn the meaning of visual terms, the prediction is wrong.

Landau and Gleitman (1985) found that blind children's acquisition of syntax, early vocabulary, and the functional uses of language can be remarkably like that of normal, sighted children. They offer compelling demonstrations that Kelli (one of their congenitally blind subjects) knew, at least by the time she was a preschooler, that sighted people can see. For example, Kelli would hold up an object when told to "let mommy see the car" and hide an object when told to "make it so mommy can't see *X*." She also turned around when asked to "let me see your back," not when asked to "let me see your front."

Kelli surely had to learn the meaning of SEE. She was not born knowing the English correspondence to this particular sound, any more than a Spanish child is born knowing that *si* means "yes." But her learning could not have taken the course that association theorists ever since Locke have assumed it must. Landau and Gleitman (1985) suggest that learning of such words occurred, in part, because Kelli *listened* to how the language she had started to learn was used by others. By listening to the way novel verbs were used in sentences, she was able to use her existing knowledge of syntax and semantics in order to construct guesses about the meaning of the visual terms. Consider how this might have happened.

To start, Landau and Gleitman (1985) note that LOOK is an agentive verb and SEE a nonagentive verb. Clauses that start with IN ORDER TO, or what are known as purposive clauses, are limited to being complements of agentive verbs. Speakers of English know they can say "John looked into the room in order to learn who was there." They also know that they cannot say "John saw into the room in order to learn who was there," presumably because they have implicit knowledge of the difference between the syntactic principles governing the difference between agentive and nonagentive verbs. There are other features about these verbs that bear on the Landau and Gleitman argument, features we leave to the reader to explore further. We turn to the points one can make with this example.

Novice learners can and do use inputs that do not fit our preconceptions of what is relevant. Had Landau and Gleitman not been willing to take seriously the possibility that the definition of relevant inputs need not be what we think it is, we might not yet know that knowledge of syntax can feed the learning of verb meanings in all language learners (Landau & Gleitman, 1985). The fact that visual concepts *can* develop in the absence of a functioning visual system highlights the need for us to reexamine assumptions regarding what counts as the foundational data for concept acquisition. We need to look for an alternative account of what the building blocks of experience are, to reconsider how one *describes* what counts as relevant inputs. Clues as to how to proceed are embedded in the previous example.

As soon as one allows that the rules of verb use can themselves serve to defi relevant inputs for different concepts, one makes a move to a structural descriptic of the required environment. No longer are the data best described in terms of wh is seen by the eyes, let alone sensed or taken in at an uninterpreted sensory leve Instead, the data are better described in terms of the principles that serve to organi the very representations the novice uses in a given learning setting. Inputs a relevant if they are structured in a way that is consistent with the organization of th principles that will assimilate them. Language learning in the deaf offers convergin evidence for these conclusions.

Until recently, it was broadly assumed that the deaf do not develop a language c their own. During the past 20 years linguists and psycholinguists have helpe revolutionize our views on this matter. It is now known that the language of th deaf, one that makes use of our manual as opposed to spoken skills, is indeed language, rich in structure for syntax, morphology, and phonetics (perhaps w should say "manetics"). American Sign Language is not a finger-spelling transla tion of English. Instead, it has its own different rules of syntax and morphology (se Klima & Bellugi, 1979; Suppalla, 1987). Of interest for this paper are Johnson an Newport's (1988) data on language acquisition in second generation congenitall deaf children, who, unlike their parents, were allowed to start learning to sign a soon as they showed any interest in language learning.

The deaf parents of the deaf children in question were raised in the oralis tradition and consequently were not exposed to sign until they went to schools fo the deaf. Even then, they did not receive instruction in sign. In-class emphasis wa on learning to speak. Nevertheless, learning of sign occurred—in the corridors, o the playing fields, in the dorms and cafeterias, and so on. Newport and he colleagues report that when the children of these same individuals are allowed to start signing as toddlers in their home, presumably with their parents serving a models, they end up with a deeper mastery of the target language than do thei parents. For example, when their parents are more likely to use frozen signs, they are more likely to take apart such complex signs and decompose them into linguistically meaningful units. The result is that they are also better able to generate novel items and acquire more advanced rules of syntax.

We have seen that our assumptions about the kind of data that many take to be relevant for first language acquisition may not mesh with the learner's assumptions. The expert's idea that the deaf cannot and do not acquire language is very much tied to a particular theory of learning. Throughout the lengthy period that we have believed that languages must be spoken, and therefore that sign language was not a language, deaf communities have presented evidence to the contrary. Yet only recently have language experts recognized the import of these data. Given our own constructivist inclinations to apply our own theories, there is no guarantee that we will avoid similar errors in the future. Similarly, it is always possible that the novice will take as relevant data that we either think are irrelevant or fail to notice. In this sense then, the novice might be said to be more expert at defining relevant inputs than are those who have already mastered the domain—a sobering thought indeed.

Constructivist theory of mind: Empiricist laws of learning? Many theorist the field of cognitive development say they prefer some variant of a nonassociat ist model. Still, it is not hard to point to many among the same set who, su unintentionally, work with one or another variant of an associationist theor environment, for example, those who focus on that aspect of the Vygots account that assigns adult caretakers a privileged status with respect to the jo knowledge transmission. In this particular interpretation of Vygotsky, adults are only the keepers and users of knowledge about concepts, social matters, cult and so on, they are also the best transmitters of this pool of knowledge. But cannot be true, at least not all the time.

We have just seen that many an expert linguist and psychologist faile discover the relevant inputs for learning about the meaning of verbs. Simil experts in other areas have made recommendations that are consistent with t theory of learning, seemingly to ignore alternatives that might make sense. example, there used to be math textbooks that taught the number facts 2 + 3 and 3 + 2 = 5 at widely separated intervals. The rationale was that this minim the possibility that children's learning would suffer from the competing and inte ing associations to common elements. Given a constructivist view of rele inputs, the preceding number facts are far from interfering. They are structu extremely similar.

Other associationist assumptions penetrate our intellectual unconscious abou conditions for learning, including that the young learn what we know sin because they are repeatedly exposed to the right environment. This explanation straightforward paraphrase of a fundamental associative law of learning, the la frequency that states that the more often a given stimulus is presented, the n novices form association networks based on the input. However, in its unmodi form, this law cannot be a constructivist law of learning. The learner need not at to the stimulation emanating from the object. And the learner could misinterpre stimulation, even if she does attend to it. In either case, across trials, the data may be common according to some objective standard. Nevertheless, it need nc common for the learner. If so, one might say that there are no repeated trials hence no inputs that are more frequent than others. In the extreme, it is possible each encounter with the "objectively" defined input is actually a different one the subject.

Another way of putting the foregoing is to say that if both the empiricist constructivist accounts of learning have a law of frequency, the law *must* wor different ways or exists for different reasons in the two cases. In the absence o nature of such a law of learning within a constructivist framework, we run the of mixing together an empiricist account of learning with a constructivist accou the mind. In fact, if a sequence of inputs is structurally related, there is no neec the exact same ones to be repeated over time. Ones that form an equivalence c because they share a structural definition might well be interchangeable, as example are 2 + 3 and 3 + 2.

Why the seeming tendency to fall back on associationist ideas about the wa nurture and advance cognitive development? Some of the tendency is surely du(the absence of a clear alternative. Given our everyday constructivist tendencies, are bound to make implicit use of whatever theory we do have. Further fact might reinforce this tendency. Historically, the associationist theory of mind : been closely tied to the democratic political view that anyone's mental repertc can be nurtured if given the right opportunities. The match between a commitm to equal opportunity and the goals of educators makes understandable our impl use of an empiricist theory of environment. It is a piece of our cultural unconsci(and therefore is used without awareness. It is as if it never occurs to us to ; whether there are other theories of the environment that are also consistent with (political and educational goals. Farfetched? We think not. In what follows we sh that there has been a strong tendency within the field of cognitive development accept as given the associationist definition of what are the first relevant data. Sir this holds for the important traditional theories of cognitive development, it is surprise that similar themes penetrate the rules for developing educational materia

Implications for Theories of Cognitive Development and Learning

The major developmental theories, be they due to Bruner, Piaget, Vygotsky, Werner, all share the premise that relevant inputs for cognitive developme proceed from first being sensory to later being abstract—from the sensorimotor the perceptual or concrete to the abstract or logical levels. Different terms are us by different theorists but all converge on the same conclusion. A similar descripti(of the stimulus is assumed by those who construct educational materials and off teachers guidelines. The theme is that one must first let children interact wi "concrete materials" and not burden them with the purely symbolic or represent tion level; they are not ready to move beyond the perceptual. We are beginning see evidence that challenges these deeply entrenched assumptions. There are no cases where the initial or early relevant inputs for cognitive development do not f the rule that learning in the young proceeds necessarily from the sensory to tl perceptual to the abstract. What has been taken as given is beginning to look mo like an assumption that is tied to a given theoretical characterization of inpi primitives.

Learning About Objects

One standard account of how infants come to know objects is that they graduall build up associations based on the sensations garnered from exposure to the object Objects that share a common color, substance, shape, texture, and so on generat(sensations that are more proximate to each other than not, and hence ones that ar most likely to be associated. Over time, the infant (or any novice learner) accrues a sufficiently rich associative store from which to induce the concept of the targe object. Similarly, the ability to find an object, even when it is partially occluded, derives from common or similar sensory associations. For example, those parts that share a color or shape should be taken as the parts that go together. Although the

Piagetian account does not treat associations as a foundational mental ability, it do share with associationism the notion that the concept of an object is built up as mo and more sensorimotor schemes are coordinated, that perception is first two dime sional, and that perception precedes conception.

An alternative to both the associationist and Piagetian accounts has been deve oped by Spelke and her collaborators. Spelke (1988) proposes that infants beg with the assumption that their environment is three dimensional and composed things that occupy space, persist, move as units independently of one another, a maintain their coherence and boundaries as they move. Two principles of obje perception follow: Two surfaces will be perceived as part of the same object if th touch each other, and two surfaces that move together at the same time and spe along parallel paths in three-dimensional space—even if their connection concealed—will be perceived as surfaces of a single object. Together these prin ples would allow infants to learn which surfaces of a partially concealed obje belong together.

Spelke's principles for defining the initially relevant data are stated in relatio or structural terms, not in terms of separate bits of sensory data. It is not becau Spelke denies infants the ability to sense these attributes; the idea is that these just not the kind of data first used. Rather, her account reverses the definition what kind of inputs are foundational and what kind of data are noticed later. particular, Spelke suggests that the patterns of light that generate the sensations color, brightness, and so on are only used after infants *first* sort the environm into things. Once noticed, infants' tendencies to explore things leads them to not and learn details about these different things. We note that an account like this better able to explain the fact that infants do not necessarily treat objects w different colors and textures on each surface as different objects (Kellman Spelke, 1983). If infants see these two surfaces moving together in parallel, they more likely to assume they belong to the same single object. Similarly, if t objects that are held, one in each hand, and connected by a *not-visible* moving r infants will treat the display as if the two objects are in fact parts of a single obj (Spelke, 1990).

Spelke's idea is that infants first pick out objects on the basis of patterns of d that are related to the nature of three-dimensional object perception. For h foundational inputs should be characterized in terms of *coordinated patterns* inputs as opposed to points of sensations. These in turn are related to the definit of what is relevant as now specified by abstract principles of perception, possibly cognition, not in terms of particular sensors that are ready to rece sensations from a given distal stimulus (Spelke, 1988).

Keeping Track of and Using Numbers at an Early Age

The possibility that infants do not treat punctate patches of light, sound wav pressure, and so on as the primitive sources of data fits with findings that show t infants respond in *numerically* relevant ways when shown representations of a *se* objects. In one experiment, six- to eight-month-old infants were shown a series heterogeneous three-item displays. On each trial each item varied in shape, col position on a screen, and so on. Similarly, item kind and a variety of sur variables of the items changed across trials. These displays were rich in sen input potential. Yet infants did not fixate on these variations in sensory informat Instead, they habituated to the class of three-item displays, as evidenced by the that they only recovered their interest in looking at displays when the nun of items contained in these changed (Starkey, Spelke, & Gelman, 1983, 1 Starkey, Gelman, & Spelke, 1985).

Infants' tendencies to respond in numerically relevant ways are robust enoug serve investigators who "ask" them other questions. For example, Kesterba Termine, and Spelke (1987) have shown that infants do not always perceive objects when adults do. When two objects of the same kind that vary only in c are side by side, so that they are touching and in perfect alignment, infants see one object; we see two objects. It is only when the objects are spatially separ that the two objects are perceived as two items by the infant.

There is an exceedingly popular and resilient belief that number-relevant sponses in infants are controlled by a low-level general purpose perceptual me nism (e.g., Shipley & Shepperson, 1990), as opposed to, say, principles of co ing. The idea is that the data do not reveal a sensitivity to number-rele dimensions but rather more primitive stimulus qualities, perhaps like brightr intensity, and so on. An example of such an account is offered by Moore, Be son, Reznick, Peterson, and Kagan (1987) for Starkey et al.'s (1983) finding infants prefer to look at that slide of a pair that contains the same number of pict as the number of drumbeat sounds they hear (two or three) at the same time. M et al. start with their finding that infants in their task responded intermodally to pair of stimuli that differed in number. They go on to say that this "failur replicate" Starkey et al. rules out a number-based account of both studies. T suggest that infants in their study responded on the basis of relative difference intensity levels. Perhaps, but the same kind of explanation does not work for Starkey et al. data because the displays within each of the two-element and th element classes of stimuli were intentionally selected to vary in color, size, an on. Therefore, there could not be a common intensity that characterized all of two-item and none of the three-item displays, or vice versa (see also Starkey et 1990).

The Moore et al. (1987) account takes particular sensory attributes as primit Like Spelke, we do not. Instead we appeal to a more relational account of relevant data. By allowing infants a principle that is akin to a principle of one- correspondence, we give them the wherewithal to detect either the presence absence of a *correspondence* between the sets of items (no matter what their sen characteristics) and end up with one account of both the Starkey et al. and Moo al. data. Why infants respond intermodally to a correspondence in the former se studies and a lack of correspondence in the latter set is not answered by this acco However, this is a different kind of question, one that awaits the answer as to infants sometimes respond intermodally to matches and sometimes to differe for a wide range of input types (Spelke, 1990).

If we allow that infants use an implicit principle of numerical correspondence, we achieve an account of their interest in, as well as attention to, inputs that are numerically relevant. The idea is that infants, like older children and adults, are motivated to apply their structures and will if the environment offers them an opportunity to do so. We also begin to zero in on the characteristics of relevant and irrelevant inputs for such nascent structures. For example, to apply a one–one principle, one simply must have two collections of entities. Otherwise, it does not matter what the entities are, including whether they are in the visual domain or not, that is, whether they project light that the retina can detect, how big they are, how bright they are individually or as a collection against a given background, what visual angle they subtend, and so on. Such characteristics are all irrelevant, *when it comes to matters of number*. We return to consider other findings that support the conclusion that infants can respond in numerically relevant ways in the section that discusses the nature of the preschooler's understanding in this domain.

Learning About Novel Instances in a Category

Despite differences in the way Bruner, Piaget, Vygotsky, and Werner discuss concept formation, they all share one theme: Young children's categories are formed first on the basis of the visible or audible or touchable surface similarities between items, for example, shape, color, size, sound, texture, and so on. Related to this conclusion is the idea that young children do not apply abstract classification structures when organizing their knowledge about objects. These conclusions have had a long staying power, most likely because they are grounded on robust and readily replicated findings (see Gelman & Baillargeon, 1983, for a review of much of the classical data). Once again, however, we challenge the fusing of results with a particular account and turn to scrutinize the belief that learning about categories necessarily starts with a focus on the sensory, that young children lack the conceptual competence to organize inputs according to hierarchies, or that young children cannot form inductions from abstract inputs. Research by S. Gelman and Markman (1986) on induction in children provides one line of pertinent data. Our own work on the role of causal principles in the development of an understanding of the animate and inanimate distinction provides a second and converging line of evidence.

S. Gelman and Markman (1986) taught four-year-olds new information about two objects and then tested to see whether perceptual characteristics as opposed to category membership determined transfer to a novel item. For example, in one study they pointed to a line drawing of a flamingo and a bat and said "this bird eats X and this bat eats Y." Ambiguous test items, for example, a gull that looks more like a bat but is an instance of the category *bird*, were used to determine whether children generalize on the basis of surface similarities or common category membership. Children's responses to test questions such as "What does this eat?" revealed a reliable tendency to generalize on the basis of category membership more than surface similarity.

Information about the category could hardly have served as a basis of generalization if the children were not inclined to interpret their environment in terms of at least some categories. That is, they must have had some conceptual competence for category structures. Evidence is accumulating that young children have considerable knowledge about a variety of category differences, including the animate-inanimate distinction (e.g., Bullock, 1985; Gelman, Spelke, & Meck, 1983; Keil, 1989; Richards & Siegler, 1986). Massey and Gelman's (1988) results serve the present discussion especially well. When the preschool-aged children in their study were asked to answer whether the item depicted in a photograph could move itself up and down a hill, items that shared surface similarity were not treated the same way. For example, unfamiliar statues of mammal-like animals were not treated as were unfamiliar mammals. Instead they were responded to in the same way that the children responded to unfamiliar wheeled objects and rigid, complex inanimate objects. Of particular interest is that unfamiliar nonmammalian animals were treated like the unfamiliar mammals, items they did not look like at all.

Of course, the children had to respond to something about the pictures. Subsequent analyses showed they used their ideas about core differences between animates and inanimates, including how they move, what they are made from, what kinds of part-whole structures they share, whether or not they can communicate, and so on, to guide their inspection and interpretation of the photographs (Gelman, 1990). Statues could not go by themselves because they were too shiny, needed a push, had no feet (even if there were feet represented in the picture), and so on. Animals could go by themselves because they were running, had feet (even when none were shown in the picture), and so on.

Inferences About Insides

Studies that ask young children about the *insides* of objects provide another important line of evidence as to whether the young are limited to using the elementary sense data given by the surface of an object. What should happen when young children try to answer when asked what is on the inside of both animate and inanimate objects? In neither case is there any reason to think they can see the insides and therefore they should not be able to offer sensible answers if they are restricted to using direct sensory data. Studies by Gelman and Meck (in Gelman, 1990) present the relevant findings.

In the Gelman and Meck studies, children between the ages of three and five years were asked to think about a series of animate (person, cat, elephant, mouse, bird) and inanimate (doll, puppet, ball, rock) items. No photographs or replicas were presented; children were simply told to think about these as they were asked in turn what was on the outside and inside of each one.

When asked the *outside* question about people, children had a strong tendency to talk about parts on the face and/or head (1.5 such answers per child). Figure 2.1 shows that questions about the *outsides* of dolls were more likely to elicit the same bias to talk about the face and parts on the face than were the outside questions for the other animate items. This result sets the stage for us to evaluate the proposal that

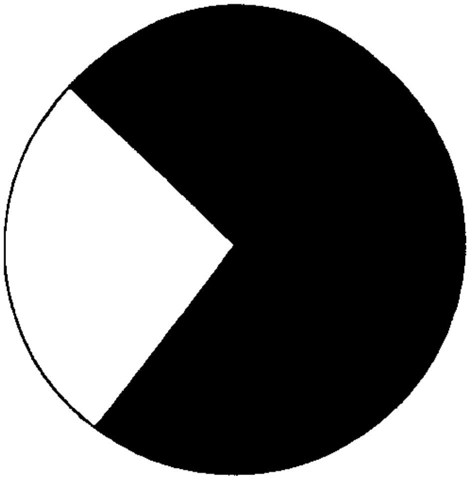


Figure 2.1. Tendency of children to say that inanimates (doll, all other relevant items) look like a person on the outside

the young children generalize on the basis of whether objects look alike on their surfaces. For example, did the children tell us that a doll and a puppet each have the same thing(s) on their respective insides as do people? Or did they, instead, treat questions about the insides of dolls and puppets more like ones about the insides of other inanimate objects, even if these did not look alike on the surface?

None of the children in any of the age groups volunteered that puppets and dolls have the same insides as do people. In contrast, despite the notable surface differences between an elephant, a bird, and a person, these same children told us that all had (at least) blood, bones, and food inside. The older the child, the more they told us of other organs on the insides of these target animate items, especially when they were answering about humans.

The blood, bones, and organs pattern of answers to the inside questions for animates was notably different from the one obtained for the inside questions about puppets and dolls. Now "insides" talk was about material (e.g., paper, cotton, fabric, stuff) "nothing" or known physical mechanisms (e.g., batteries, strings, etc.). Further, whereas the inside answers across animate types were relatively homogeneous, answers across inanimate types were decidedly heterogeneous. Children who knew something about the object in question knew to answer about the specific material or mechanism that might be inside that object. When they did not, they often answered in ways that suggested they thought the same features characterized the insides and outsides, for example, when they said steel (patterns) was on both the outside and inside of a ball or that cotton was on both the inside and outside of a doll.

These findings are reinforced by additional studies appearing in the literature (S. Gelman & Wellman, 1991; Markman, 1989). More and more it begins to look like we may have underestimated the young child's ability to reason abstractly and to form inductions from nonsensory, nonconcrete data. We return to this conclusion in the final section on our work on fractions.

Implicit Knowledge About Number Goes to School

In our discussion of how to characterize inputs within a constructivist account of learning, we introduced evidence of infants attending to number-relevant inputs. Other studies add weight to these findings. Strauss and Curtis (1984) have shown that infants respond to numerical orderings and Sophian and Adams (1987) have demonstrated that infants keep track of the numerical effects resulting from addition and subtraction. Such findings support our idea that early skeletons of domain-specific principles are available to support interest in and learning about number. So does the fact that young children enter school with a principled understanding of counting and its relationship to adding and subtracting (within a limited range of integers).

By about four or five years of age, children are able to count in principled ways, invent solutions to novel counting problems, detect errors in counting trails generated by others, and make up counting algorithms to solve simple addition and subtraction problems, at least for a limited range of numbers. (For reviews see Fuson, 1988; Resnick, 1988; Gelman & Greeno, 1989). It is not just preschool children who are able to take advantage of their environment to develop their skeletal knowledge of numbers. All over the world, both young children and nonschooled adults use counting algorithms to solve arithmetic problems. These are often made up and resemble those that schooled children invent.

Whether schooled or unschooled, individuals have strong tendencies to decompose natural numbers into manageable or known components, to count when adding or subtracting, and to use repeated addition (or subtraction) to solve "multiplication" (or "division") problems. These multiplication and division solutions are used both before and after children have been taught more standard multiplication and division algorithms in school. They are also used by unschooled adults and children in a variety of settings and cultures (e.g., Carraher, Carraher, & Schliemann, 1985; Ginsburg, 1977; Groen & Resnick, 1977; Lave, 1988; Resnick, 1986 Saxe, 1988; Saxe, Guberman, & Gearhart, 1987; Starkey & Gelman, 1982).

Some of the algorithms invented by older children and unschooled adults an more complex than those used by preschool children. Still, as Resnick (1986) note in her analysis of these, all use principles of counting and addition (or subtraction with the positive integers. An example from her own work illustrates this. Pitt (yrs–7 mos) said that he solved "two times three" as follows: ". . *two threes . . one three is three, one more equals six*." Similarly, schooled and unschool individuals in Africa and Latin America use a combination of number decompos tion moves and repeated additions (or subtractions) to solve the multiplicati problems presented by investigators. Young children are also less able to work wi large numbers; otherwise, it is hard to distinguish their invented, out-of-sch solutions from those generated by older children and adults (Carraher et al., 1985; Saxe, 1988). Lave, Murtaugh, and de la Rocha's (1984) work with shoppers in a California supermarket provides an especially compelling documentation of how everyday "intuitive" solutions for determining unit prices are preferred over any taught in school.

This widespread invention of algorithms that are based on counting and/or repeated addition (subtraction) algorithms provides further support for our conclusion that a universal set of implicit principles governs the acquisition of initial mathematical concepts (Gelman, 1982; Gelman & Meck, 1986). A skeletal set of counting principles, in conjunction with some principles of addition and subtraction, promotes the uptake of mathematical data that are relevant to these principles. The skeletal principles provide the a priori structure necessary for learners to notice, assimilate, and store relevant data in an organized manner (Gelman, 1990; Gelman & Greeno, 1989).

The obvious question is whether this kind of knowledge provides a ready base upon which to achieve the goals of mathematics teachers—to teach mathematics. On the assumption that learners will be inclined to apply their initial theory about numbers to the novel inputs offered in schools, it is not clear that the knowledge they bring to the setting will support the correct interpretation of some kinds of data presented in math classes. To illustrate why, we consider some of our work on how kindergarten, first-grade, and second-grade children interpret commonly used teaching materials, so common in fact that it might be hard to believe they are misinterpreted. We will see that although preinstructional assumptions about the nature of numbers may serve many in their everyday interactions, they may nevertheless interfere with the learning of mathematics taught in school, even the seemingly simple concept of a fraction.

Learning About Fractions Might Be Hard, Despite Repeated Exposure to "Relevant" Inputs

From early on, children are repeatedly exposed to inputs that are assumed to be relevant for learning that fractions are kinds of numbers. These include the number line, fractional parts and wholes of circles, squares, rectangles, and triangles, as well as the numeric and written forms of ½, ¼, and ⅓. If decimal notation is introduced it is in the context of lessons about money. Visitors to American classrooms for Kindergarten, Grade 1, and Grade 2 children will see the number line as well as parts and wholes of shapes on the classroom's walls and blackboards more often than they will find symbolic forms of fractions in textbooks and notebooks. The former are frequently described by educators as concrete, the latter as abstract (cf. previous discussion on this distinction).

Although written fractions involve abstract symbols (be these numerals or words), the number line and part–whole shape items are *not* concrete in the usual sense. They too are representations of number in this context. On their own, they afford many other interpretations, for example, a line, a pie, a circle, round, and so on. If they are truly concrete materials, there is no reason to assume that the child will get the intended interpretations, let alone think of these in terms of fractional numbers. For this reason, it is important to ask how young children actually interpret such common classroom props. What mathematical concepts do they think these stand for?

Although there are principles that both guide and structure early learning about counting, this could be a mixed blessing. Whatever the mathematical prowess of the young child, further learning often requires that one transcend the principles that guided early learning. Much of mathematics involves operations and entities other than counting and the addition and subtraction of the counting numbers (or the positive integers). When it comes time to go beyond the early knowledge that is built upon the counting principles, continued adherence to these principles could yield misinterpretations of inputs designed to foster new learning. Given a constructivist mind, this seems likely if the data are not relevant (from the expert's viewpoint) to the count numbers. To illustrate why, we need not go too deeply into mathematics. The idea that a fraction is a noncounting number, but nevertheless a number, will serve our purposes—especially since it seems to be a watershed in elementary school mathematics learning (e.g., Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980).

One can characterize the way numbers are first thought about as follows: *Numbers are what you get when you count things and combine or take apart counts and/or their cardinal representations*. If this is a true characterization, what should happen when a child with this theory (be it implicit or explicit) attempts to assimilate common classroom inputs for learning about fractions? One cannot count things to get the answer to "Which is more ½ or ¼; 1.5 or 1.0?" Similarly, number lines are not simple representations of whole numbers. Young children might know that one can take apart a circle and a rectangle and get two "halves" on both occasions and still not appreciate why each of these halves can be represented with the written expression ½. The interpretation of the latter might be assimilated to the idea that such marks on paper must be about whole numbers.

In the absence of implicit principles for dealing with fractions, young learners might "overgeneralize" their counting principles and produce a distorted assimilation of the instructional materials on fractions. For their theory cannot handle such data veridically; it lacks principles that can recognize the input for what it is. This is because fractions are numbers generated by the division of two numerosities; *they are not count-numbers*. Since there is no reason to presume that the requisite principles for dealing with such entities are available when they are introduced, there is every reason to expect young children to err in their interpretation of data designed to teach them about fractions.

Studies have found children in the middle- and high-school grades make systematic errors with rational numbers (e.g., Behr, Washsmuth, Post, & Lesh, 1984; Hiebert & Wearne, 1986; Kerslake, 1986; Nesher & Peled, 1986). We know of none that have targeted children in their first few years of school, presumably because early math instruction does not focus on this topic. We thought it was especially important to focus on young children. If still younger children do brin with them the idea that numbers are what one gets when one counts things, they might start building, at an early age, erroneous representations of data meant to exemplify alternative notions of what numbers are about. These representations could, in turn, stand in the way of children's correct interpretation of later lesson plans on fractions, no matter how frequently they are presented.

As indicated, children in the United States in Kindergarten, Grade 1, and Grade 2 encounter fraction-relevant material and receive some instruction about fractions. Since we interviewed a sample from each grade at the end of their school year, even the kindergarten children had experienced at least the number line and the various symbolic forms (spoken terms, written words, and written numeric expressions) for ½, ¼, and ⅓. Some also were introduced to appropriately labeled and marked measuring cups in cooking class. The children in the first and second grades had more experiences like these, both in terms of classroom presentations and testing opportunities. Therefore, although their curricula did not delve into the conceptual and arithmetic characteristics of fractions as numbers, the children were offered some relevant data about the nature of fractions. How do these children interpret such offerings?

Fractions in the early grades

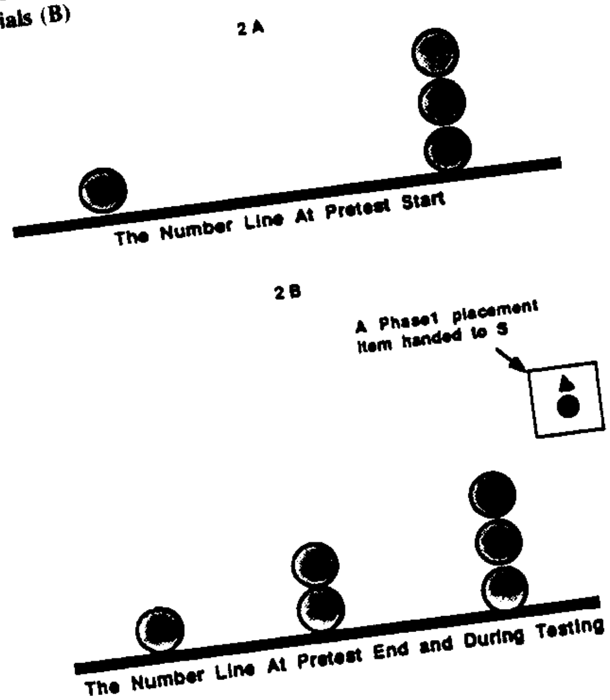
Some details about the study. The kindergarten (N = 16, mean age 5 years–9 mos), first grade (N = 12, mean age 6 yrs–9 mos), and a second grade (N = 12, mean age 7 yrs–9 mos) children who participated in the study attended one of three schools in the Greater Philadelphia area, all of which drew from middle-class samples. We had signed permission from the parents to interview the children in a quiet room away from their classrooms. The interviews were conducted on three different days. The first and second days of the interview were separated by at least two days (but not more than a week). In almost all cases the third day of the interview could occur anywhere between one and two months after the second.

The study involved a pretest, a five-phase placement interview, and a follow-up battery of arithmetic items. The sequence of the placement phases was designed to provide more and more task-relevant information without giving specific answers. Brown and Reeve (1987) recommend the use of such progressively more explicitly "hinting" in order to bring out whatever competence a child might have. Our own work confirms their view that such hints serve to limit misunderstanding about the task and therefore false negative attributions by an investigator (see also Gelman, 1978; Gelman & Meck, 1986). The items for the first two phases of our placement sequence were presented without hints so as to allow us to obtain some baseline data. Each successive phase after these introduced more and more relevant mathematical information and offered more detailed mathematical descriptions about the props in the task. This meant that we provided successively more hints about the nature of the task as we moved to the use of more and more explicit mathematical language.

The pretest questions and interactions familiarized children with "our special number line." When children came into the room, they saw "The Count," a puppet from the American televised program *Sesame Street*, sitting in the middle of the table alongside the folded-up number line. They were told that The Count had come to visit them at school because he wanted to learn new things about numbers.

To start the pretest, a child was shown, one at a time, displays of 1½, ½, 1¼, ¼, and ⅓ circles and asked "how much" (or "how many" since children offered this alternative) were present. If a child could not name 1½ or 1¼, the experimenter pointed to the whole circle and said, "This is one circle." Then, while pointing to a part of a circle (½ or ¼), she asked "What's this?" The part was correctly identified for children who could not answer on their own. After this introduction to relevant terms, the experimenter unfolded her "special number line" schematized in Figure 2.2A. The line was intentionally long (4′4″) so as to give the impression that it went on and on in both directions. Instead of using numerals to represent the cardinal values for successive integers, we used a line that had sets of *N* circles (4.75″ in diameter) where the integers should have been. At first a child saw

Figure 2.2. Schematic representations of the "Special Number Line" as seen by a child at the start of pretesting (A) and during placement trials (B)



representations of 1 and 3; it was his task to take a set of two circles and place it where it should go, to answer what number would come after one and before three.

The pretest continued with talk about the ordering relations between the whole numbers, for example, "Is 4 more than 3, less than 3, or the same as 3; is 2 more than 1, less than 1 or the same as 1?" If necessary, feedback was provided since we wanted to encourage children to think of ordering numerical values. Then, to end the pretest, children were told, "The number line shows the numbers in order." To show us that they knew "how our number line works," they were then asked to place sets of *N* (1, 2, 3, or 4) small circles (ones much smaller than those on the line) "where they belong."

The pretest phase was followed on the same day by some of the phases of a five-phase fraction placement sequence and related test items. A sample fraction placement trial is shown in Figure 2.2A. On such trials a child's task was to put each test display below the point on the line "where it belonged" and then explain why it went there. No hints were provided for any of the test items used during either of the first two placement phases. In Phase 1, test displays contained 1½, 1⅓, and 1¼ circles. The second phase test displays contained ½, ⅓, and ¼ of a circle. Once Phase 2 was done, children were asked how the test items for that phase were the same or different. Although no hints were provided for the test items used during the first two phases, if children erred on either 1½ in Phase 1 or ½ in Phase 2, these items were repeated at the end of their respective testing periods, and the children were asked whether the numerical value represented was equal to, more than, or less than a whole number value.

The experimenter started Phase 3 by placing ½ circle between the positions for 0 and 1, and saying "we can put one half between zero and one because it is more than zero and less than one." A similar trial followed with a display with 1½ circles. Finally, the child was told, "We can count ½, 1, 1½, 2." Then the child was asked to place the test items (2½, 1¼, ⅓) and explain their placement. The experimenter did *not* verbalize the corresponding numerical values for these test items. The experimenter resumed talking about number at the end of the Phase 3 part of the placement interview. No props were used for this. A child was simply asked whether there were numbers between 1 and 2 (or 0 and 1), and if so how many there were. If a child said that there were no numbers between the named integers, the experimenter went on to task if 1½ (or ½) was such a number. The opportunity to count by rote and solve *verbally* presented arithmetic problems ended the interview for the first day.

Phase 4, which occurred at least two days, and as much as a week, after the previous placement phase, started with a brief reminder about the "special number line." For all subsequent test and demonstration items, the experimenter now used the terms that corresponded to the numerical values instantiated by the displays when she presented these. Children were questioned extensively about the ordering relations of the values represented by the described test items as well as the ordered positions the corresponding stimuli should assume on the number line. Phase 5 was much like Phase 4 and came right after it. These two phases differed mainly in terms of the display values presented.

The end of Phase 5 included some items designed to assess the extent to which our subjects interpreted the task as one that had something to do with counting things. A separate follow-up phase, a month to two later, yielded data on how the children read noninteger numerals when each was presented separately on a card. Finally, the design included some arithmetic items so that we could assess whether there is a relationship between our assessments of arithmetic skill and level of success on the fraction placement task. For example, we asked children to solve simple arithmetic problems with values of 0 and ½. (For more details, see Table 1 of Gelman, Cohen, & Hartnett, 1989.)

Some results

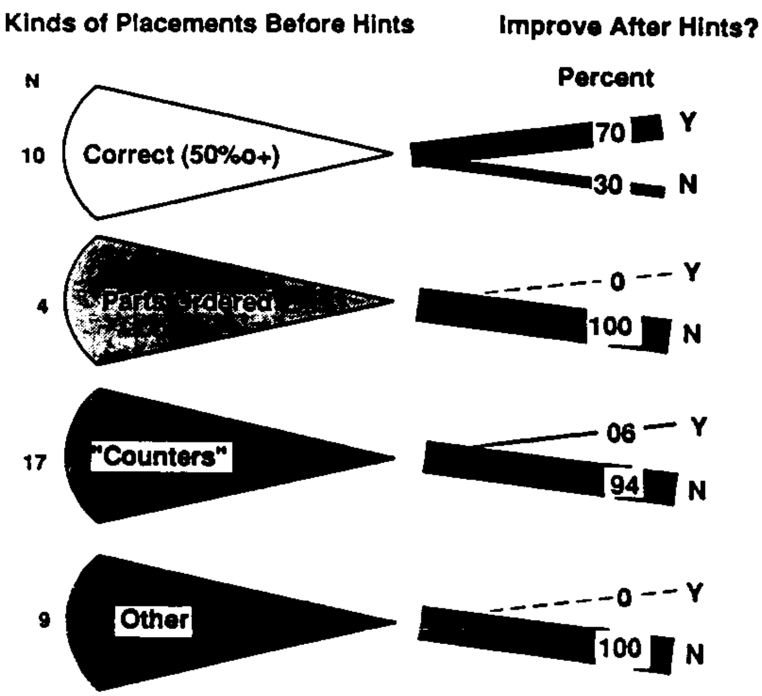
Kinds of baseline responses. Responses to the fraction placement target items were coded for each phase of the testing. Inspection of individual patterns of responding across Phases 1 and 2 (i.e., before hints were offered) revealed four patterns of baseline responding:

1. *Correct (at least 50%).* Children who used this baseline pattern of responding placed their items so as to integrate a metrically ordered positioning of the fractional parts and whole circles without feedback and did so on at least 3 of their total (6) Phase 1 and 2 test trials.
2. *Parts Alone Rank Ordered.* Some children neglected any whole circles in a display and responded as if they simply rank ordered the relative sizes of the parts. For example, one child placed 1¼ at "1," 1⅓ at "2," and 1½ between "2" and "3." Such responses map relative amount of area to relative length without regard to the size of the interval between successive points on the number line.
3. *Whole Number Placements.* Children assigned to this baseline category used counting strategies, either to count the number of separable parts and place accordingly, for example, 1½ circles at "2" and all of ½, ⅓, and ¼ circles at "1," or to ignore the fractions of a circle and simply count the remaining whole circles to place the former stimuli at either "1" or "0."
4. *Others.* All remaining response patterns were coded in this category. For Phases 1 and 2 these included those where children placed successive test displays from left to right, put each test item at a different position without any concern for order, or generated sequences that we could not decode. Once the experimenter began to show children where to put displays containing one half of a circle (during Phase 3), some started to mimic her. Mimics simply placed nearly all displays that had *any* parts on them halfway between two whole number positions. That is, they even placed displays containing ⅓ and ¼ at half-way points between successive instantiations of whole numbers.

The effect of hinting. Only 25% of the children responded according to baseline placement pattern (a), that is, correctly on at least 50% of their Phase 1 and Phase 2 test trials. A full 44.5% used counting strategies. A small number of the remaining children chose to ignore the whole circles when they encountered a combination of a part and one or more wholes. For such stimuli they simply rank ordered their relative size along three different points on the number line. Finally, there were a group of children who interpreted the task in nonnumerical ways, for example, they placed each successive test item further and further along the line.

Given these qualitatively different ways the children first interpreted the number line task, it is possible that the different baseline groups would likewise interpret the hints that followed in very different ways. Figure 2.3 shows that, depending on which baseline pattern a child started out with, the hints had opposite effects. If children started out being correct on at least 50% of their baseline test trials, they benefited from the hints. In contrast, if children started out as counters or in any of the other groups, they did not. Instead, they actually tended to get worse over trials. One reason for the deterioration in their performance has to do with their tendency to mimic the experimenter's demonstrations with halves. Given a hint that they

Figure 2.3. Differential effect of hinting as a function of children's baseline placement abilities as assessed during Phases 1 and 2



went between whole number places, mimickers placed all items with fractions of a circle on them at the *same* place trial after trial, that is, a halfway point, or some other constant point between two whole numbers.

The data shown in Figure 2.3 summarize the two main findings reported thus far. First, young children do have a strong bias to interpret the positions on a number line as if they represent whole numbers, a decidedly wrong conception. Second, didactic hints from experts had a differential effect depending on how well children did according to the baseline assessment. Those who tended to share out interpretation of the representations involved benefited from talk about fractions, ordering relations, and the like. Those who did not start out this way did not benefit. If anything, our inputs served to depress their performance levels, possibly because our talk offered children a clue that they were to do something else but no assimilable clues as to what to do. In other words, their conceptions stood in the way of their interpreting the inputs we hoped would help them.

The latter finding shows that most children in our study ran their initial theory of number roughshod over the inputs that are commonly used to introduce conceptions of fractions. This suggestion is supported by further results of the study. For example, we asked children to read aloud what ½ and ¼ said when each was printed on a card. Although the vast majority of the children could "point to the half" (our request), only those children who met the correct baseline placement criteria came within any range of accuracy when asked to read the numeral representations of the fractions. These children said things like "a half, one fourth" or "one and a half" (which is wrong), "one fourth," and so on in response to our request that they read the fractions. Children in the other three baseline groups had a terrible time reading these in mathematically correct ways; they offered a variety of novel answers, including "1 and 2"; "1 and 4" or "1 line 2," "1 line 4"; or "1, 2" and "1, 4" or "1 plus 2" and "1 plus 4." Some even offered the answers of 3 and 5, the sum of the two numerals, or 12 and 14, a totally unexpected interpretation of the two numerals in the fraction. In fact, only 5% of all of the children in the three noncorrect baseline groups read the fractions correctly.

Reading, comparing, and adding. Similarly tendencies to assimilate the input to the idea that representational materials are to be interpreted as if they are about whole numbers comes from results for the "which is more" items. Those in the three "incorrect" baseline placement groups had a reliable bias to select the reciprocals with the larger denominators as more for both the ¼ vs. ½ and 1/56 vs. 1/76 pairs of numerals (x^2_1 = 11.27, p < .001 and 8.06, p < .01, respectively). Although children in the correct baseline placement group did not have such a bias, they did not get the items right either. They simply had no bias and performed at chance levels.

Summing up. In sum, our guess that young children would interpret inputs designed to teach about fractions as yet more data to which to apply their initial theory of what a number is, the idea that numbers are what you get when you count entities, was supported. Simply making these materials available on a repeatable basis does not suffice to lead children to treat them as items that can be interpreted in a new way. In fact, reports by other investigators lead to the conclusion that additional schooling may serve mainly to provide yet more grist for this first theory of number and thereby increase the number of misunderstandings or misconceptions children end up with in their mathematics classes.

These findings merge with ones from related research with older children in the higher elementary grades. Here too students have a tendency to inappropriately generalize what they learn about the integers to other numbers. For example, they seem to believe that they can assume that the product that results when one multiplies two numbers is always larger than either of the original numbers—even when the numbers are fractions (e.g., Greer, 1987; Hart, 1981). Although this principle is valid for the *addition* of positive rationals, it is not for their multiplication. In fact, many secondary school students lack a mathematical understanding of division and multiplication (e.g., Fischbein, Deri, Nello, & Marino, 1985; Vergnaud, 1983), leading one to wonder how they could possibly understand that fractions are noninteger numbers. Findings like these make it clear that it is not only the young child's spontaneous theory of number that is exceedingly robust. It seems that a very large number of children continue to interpret inputs about fractions as if these were examples of integer numbers.

A Suggestion: To Learn Mathematics Is to Learn the Language and Structure of Mathematics

Despite the foregoing, we should not loose track of an equally important set of results from the studies already cited. Some children do rather well, that is, some children move on to more advanced mathematical interpretations given the opportunity to do so. How? Are there any clues to turn to either from research or our discussion of a constructivist theory of learning? We think there may be.

Recall that children's ability to correctly interpret one representational format of number, for example, our number line, went hand in hand with their ability to work correctly with the symbolic representations of the entities, operations, and principles of arithmetic. A similar result obtains in a subsequent questionnaire study we did with children in Grades 2 through 8 (Gelman, Cohen, & Hartnett, 1989). Many in this group responded incorrectly, much as did the young children who erred in our first study. However, some children in this second study were well on their way to a principled understanding of fractions as numbers. The data in Table 2.1 serve to illustrate.

Table 2.1 presents data from two grade levels as well as two ability levels. Children in the gifted classes in the school system visited are assigned to this track early on (sometimes Grade 2) on the basis of overall test scores. Therefore they may or may not have unusual mathematical talents. It is noteworthy that many of the answers offered by children in both the regular fourth- and seventh-grade classes can be characterized as tautological. In contrast, the gifted fourth-grade pupils

Table 2.1. Samples of Written Answers to "Why Are There Two Numbers in a Fraction?"

Grade and Subject Number

Regular 4th and 5th Grade

1. "Because if there weren't 2 numbers then you couldn't have a fraction."
2. "It is two numbers"
3. "Can't explain"
4. "To be equivalent."
5. "Because a fraction is a part of something. Not the hole thing."
6. "Because you have to hav a denomonator and a numarator"
7. "1 for one certain color on the thing 4 for all the things in the group"

Gifted 4th and 5th Grade

6. "4 is how many piecies and 1 is how many you got"
8. "the bottom one is how many in the whole and the top one is how many are left a in the whole."
9. "The denmunator is the hole, the numirator is how many peices you have of the hole"
10. ". . . one of the numbers stand for how many thimes something is divided up into and th other how many are taken."

Regular 7th Grade

11. "Because they wanted it that way."
12. "1/4 1 explains how many you have. 4 explains how many there are"
13. "1 shows you how many you have an the other shows you how many are there"
14. "Because 1 number is how many of something you have and how many there are in a whole."
16. "Because It Broke in to fractions I GUESS"
17. "Because"
18. "because it is a portion of a number"
20. "you need a numberator & denominator"
21. "the top one explain how many of the Bottom one there are"
24. "It is less than one whole"
25. "to show both halfs"
26. "Because it shows how many items you have out of a given amount of numbers."

provided rather acceptable answers to our novel question. These same children outperformed their grade-level peers and the regular seventh-grade children on (items as well. Whatever the locus of the gifted effect, it is clear that these chil are able to read and write about mathematical entities in at least a quasi-princi] if not principled way—even though we know they were not taught the answer tc particular question.

The latter finding fits with one previously presented on the relationship bet level of mathematical understanding and language use—that they are positi correlated. Could it be that the dictum that "one only understands something (i.e., explicitly) if one can talk or write about it clearly" is based on a pedago principle after all? Might it be that the engine that drives development in t children who do move on to learn more mathematics is decorated with material the relevant language—that the way to characterize the learning that does take] in some children is to think of it as something akin to second language learning? If so, it occurs to us that we could be erring in putting off the teaching of such matters when it comes to the teaching of mathematics and science. One cannot be a fluent user of any language without an opportunity to master both the vocabulary and syntax of the domain in question. And it is at least common wisdom that immersion opportunities are better able to support inductions than are those that introduce a language bit by bit over many years.

True, to move in this direction would mean to go against the tide, the strong commitment to the view that we should start with the concrete. But, since there is nothing concrete about the language and principles of mathematics, it is not clear what it means to do this. Blocks or rods on their own are not mathematical objects, any more than they are props for a physics experiment. To become mathematical, they have to be interpreted, and not necessarily in the same way as they have to be interpreted, and not necessarily in the same way as they have to be interpreted in a physics experiment. They might be a representation of a given number in a mathematical setting and devices for balancing in a physics context. In other words, given the settings involved, physical items do not on their own afford their interpretations. Labels, concepts, symbols, and the like serve this purpose. And to the extent that these labels are understood, the object themselves become representations or entities within the domain in question. Sticks are number lines because they represent number; blocks are entities in a balance experiment because of their physical characteristics. In short, objects "out there" have to be interpreted by the mind if they are to stand for mathematical terms or concepts. Withholding such descriptions might amount to withholding data that are relevant. Similarly withholding information about mathematical operations and the relations between them might amount to withholding the data needed to develop new skeletal structures to which to assimilate new data.

The emerging data line in preschoolers shows that they can reach inductions from abstract inputs. Still, the hard work remains, this being to characterize the form of inputs that might succeed given the constructivist inclinations of the child, one who we already knows has a sufficiently robust theory of number that she might resist any talk about different ways of thinking about numbers, be it couched in the language of mathematics or not. But one thing seems clear. A failure to try to structure input shortchanges children. For they do not even get the database necessary for the correct induction. Perhaps we owe it to the children to try to offer data designed to provide structured inputs and the related structures to which to assimilate these. The idea is not that we fill blackboards full of symbols, but rather that we insist on talking about the kind of things we teach in ways that bring out the relevant mathematical structures and how these relate to the kinds of mathematical entities pupils encounter. Lampert's (1986) work on the teaching of multiplication offers us one kind of existence proof that this can be done and can be effective. Math class dialogues between the Japanese teacher and his first-grade pupils offer an example of another kind (Stigler, 1988).

Even if we adopt the preliminary ideas in this section and proceed to develop suitable materials, we are well advised to proceed with caution. For no matter how consistent the recommendations may be with the kind of constructivist learning principles I have been trying to develop, there is one bold fact to remember. In the end, the children will determine whether we are right. They may treat our new ideas about relevant inputs as they have treated ones that come before—not consistent with either how they interpret the data or how they actually induce new principles. Or they may approximate a shared interpretation of some inputs and reject others. In other words, they must always remain our final source of validation and input as to how to tinker with, alter, or even change their curriculum.

For those who think that this caution is far from an optimistic way to end this chapter, we have but one rejoinder. In our efforts to develop physics materials for a museum for young children, we failed at first. After watching the children, we changed the exhibit and made some progress, but not the kind we wanted. A third trip to the drawing boards followed. This time we succeeded, that is, we ended up with an exhibit in which children talked about physics-relevant things to do. It is noteworthy that here too we had to introduce relevant talk—in this case, talk that pertains to scientific experimenting (Gelman, Massey, & McManus, 1989).

References

Behr, M. J., Washsmuth, I., Post, T. R., & Lesh, R. (1984). Order and equivalence: A clinical teaching experiment. *Journal for Research in Mathematics Education, 15*(5), 323–341.

Brown, A., & Reeve, R. A. (1987). Bandwidths of competence. The role of supportive contexts in learning and development. In L. S. Liben & D. H. Feldman (Eds.), *Development and learning: Conflict or congruence*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Bullock, M. (1985). Animism in childhood thinking: A new look at an old question. *Developmental Psychology, 21*, 217–225.

Carey, S. (1986). Acquisition of scientific knowledge: The problem of reorganization. *American Psychologist, 41*(10), 1123–1130.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Jr., Lindquist, M., & Reys, R. E. (1980). National assessment: Prospective of student's mastery of basic skills. In M. Lindquist (Ed.), *Selected issues in mathematics education*. Berkeley, CA: McCutchen.

Carraher, T. N., Carraher, D. W., & Schlieman, A. D. (1985). Mathematics in the street and in school. *British Journal of Developmental Psychology, 3*, 21–29.

Feldman, H., & Gelman, R. (1987). Otitis media and cognitive development. In J. F. Kavanagh (Ed.), *Otitis media and child development* (pp. 27–41). Parkton, MD: York Press.

Fischbein, E., Deri, M., Nello, M., & Marino, M. (1985). The role of implicit models in solving problems in multiplication and division. *Journal of Research in Mathematics Education, 16*, 3–17.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Gelman, R. (1978). Cognitive development. *Annual Review of Psychology, 29*, 297–332.

Gelman, R. (1982). Basic numerical abilities. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 1, pp. 181–205). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Gelman, R. (1990). First principles organize attention to relevant data and the acquisition of numerical and causal concepts. *Cognitive Science, 14*, 96–106.

Gelman, R., & Baillargeon, R. (1983). A review of some Piagetian concepts. In J. H. Flavell & E. M. Markman (Eds.), *Cognitive development* (pp. 169–230). New York: Wiley.

Gelman, R., & Brown, A. L. (1986). Changing views of cognitive competence in the young. In N. Smelser & D. Gerstein (Eds.), *Discoveries and trends in behavioral and social science* (pp. 175–207). Commission on Behavioral and Social Sciences and Education. Washington, DC: NRC Press.

Gelman, R., & Cohen, M. (1988). Qualitative differences in the way Down's Syndrome and normal children solve a novel counting problem. In L. Nadel (Ed.), *The psychobiology of Down's Syndrome* (pp. 2–99). Cambridge, MA: MIT Press/Bradford Books.

Gelman, R., Cohen, M., & Hartnett, P. (1989). To know mathematics is to go beyond the belief that "Fractions are not numbers." (Report No. CSRP-90.5). Los Angeles, CA: UCLA Cognitive Science.

Gelman, R., & Greeno, J. (1989). On the nature of competence: Principles for understanding in a domain. In L. B. Resnick (Ed.), *Knowing and learning: Essays in honor of Robert Glaser* (pp. 125–186). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Gelman, R., Massey, C., & McManus, M. (1989). Characteristics supporting environments for cognitive development: Learning from children in a museum. In J. M. Levine & L. B. Resnick (Eds.), *Socially shared cognition.* Washington, DC: American Psychological Association.

Gelman, R., & Meck, E. (1986). The notion of principle: The case of counting. In J. Hiebert (Ed.), *The relationship between procedural and conceptual competence* (pp. 29–57). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Gelman, R., Spelke, E., & Meck, E. (1983). What preschoolers know about animate and inanimate objects. In D. Rogers & J. A. Sloboda (Eds.), *The acquisition of symbolic skills* (pp. 297–326). London: Plenum.

Gelman, S., & Markman, E. (1986). Categories and induction in young children. *Cognition, 23,* 183–209.

Gelman, S., & Wellman, H. M. (1991). Insides and essences: Early understandings of the nonobvious. *Cognition, 38,* 213–244.

Ginsburg, H. P. (1977). *Children's arithmetic.* New York: Van Nostrand.

Glaser, R. (1987). Teaching expert novices. *Educational Researcher, 16,* 5.

Greer, B. (1987). Nonconservation of multiplication and division involving decimals. *Journal for Research in Mathematics Education, 18,* 37–45.

Groen, G., & Resnick, L. B. (1977). Can preschool children invent addition algorithms? *Journal of Educational Psychology, 69,* 645–652.

Hart, K. (1981). *Children's understanding of mathematics.* London: John Murray.

Hiebert, J., & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Johnson, J. S., & Newport, E. L. (1989). Critical period effects in second language learning: The influence of maturational state on the acquisition of English as a second language. *Cognitive Psychology, 21,* 60–90.

Keil, F. (1989). *Concepts, word meanings and cognitive development.* Cambridge: MIT Press.

Kellman, P., & Spelke, E. (1983). Perception of partly occluded objects in infancy. *Child Development, 57,* 72–86.

Kerslake, D. (1986). *Fractions: Children's strategies and errors. A report of the strategies and errors in the secondary mathematics project.* Windson, England: Nfer-Nelson.

Kesterbaum, R., Termine, N., & Spelke, E. S. (1987). Perception of objects and object boundaries by three-month-old infants. *British Journal of Developmental Psychology, 5,* 367–383.

Klima, E. S., & Bellugi, U. (1979). *The signs of language.* Cambridge, MA: Harvard University Press.

Lampert, M. (1986). Knowing, doing and teaching multiplication. *Cognition and Instruction, 3,* 305–342.

Landau, B., & Gleitman, L. (1985). *Language and experience: Evidence from the blind child.* Cambridge, MA: Harvard University Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* New York: Cambridge University Press.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). Dialective of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 67–94). Cambridge, MA: Harvard University Press.

Markman, E. M. (1989). *Categorization and naming in children.* Cambridge, MA: MIT/Bradford Press.

Massey, C., & Gelman, R. (1988). Preschoolers decide whether pictured unfamiliar objects can move themselves. *Developmental Psychology, 24*, 307–317.

Moore, D., Benenson, J., Reznick, S., Peerson, M., & Kagan, J. (1987). Effect of auditory numerical information on infants' looking behavior: Contradictory evidence. *Developmental Psychology, 23*, 665–670.

Nesher, P., & Peled, I. (1986). Shift in reasoning. *Educational Studies in Mathematics, 17*, 67–79.

Resnick, L. B. (1986). The development of mathematical intuition. In M. Perlmutter (Ed.), *Perspectives on intellectual development: The Minnesota Symposia on Child Psychology* (Vol. 19, pp. 159–194). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Resnick, L. B. (1988). Learning in school and out. *Educational Researcher, 16*, 13–20.

Richards, D. D., & Siegler, R. S. (1986). Children's understandings of the attributes of life. *Journal of Experimental Child Psychology, 42*, 1–22.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Research, 17*, 14–21.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early development. *Monographs of the Society for the Research in Child Development, 52* (Serial No. 216).

Shipley, E., & Shepperson, B. (1990). Countable entities: Developmental challenges. *Cognition, 34*, 109–136.

Sophian, C., & Adams, N. (1987). Infants' understanding of numerical transformations. *British Journal of Developmental Psychology, 5*, 257–264.

Spelke, E. (1988). Where perceiving ends and thinking begins: The apprehension of objects in infancy. In A. Yonas (Ed.), *Perceptual development in infancy. Minnesota Symposium on child Psychology* (Vol. 20). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Spelke, E. (1990). Principles of object perception. *Cognitive Psychology, 14*, 29–56.

Starkey, P., & Gelman, R. (1982). The development of addition and subtraction abilities prior to formal schooling in arithmetic. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Cognitive development: Vol 3. Handbook of child development* (pp. 99–116). New York: Wiley.

Starkey, P., Gelman, R., & Spelke, E. S. (1985). Response to Davis, Albert, & Baron's detection of number or numerousness by human infants. *Science, 228*, 1222–1223.

Starkey, P., Spelke, E. S., & Gelman, R. (1983). Detection of intermodal correspondence by human infants. *Science, 222*, 179–181.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 34*, 97–127.

Stigler, J. W. (1988). The use of verbal explanation in Japanese and American classrooms. *The Arithmetic Journal, 36*(2), 27–29.

Strauss, M. S., & Curtis, L. E. (1984). Development of numerical concepts in infancy. In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 131–155). Hillsdale, NJ: Lawrence Erlbaum Associates Inc.

Suppalla, T. (1987). *Structure and acquisition of verbs of motion and location in American Sign Language*. Cambridge, MA: MIT/Bradford.

Vergnaud, G. A. (1983). Multiplicative structures. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 128–175). New York: Academic Press.