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Visual nesting impacts approximate number system estimation

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Abstract The approximate number system (ANS) allows people to quickly but inaccurately enumerate large sets without counting. One popular account of the ANS is known as the accumulator model. This model posits that the ANS acts analogously to a graduated cylinder to which one "cup" is added for each item in the set, with set numerosity read from the "height" of the cylinder. Under this model, one would predict that if all the to-be-enumerated items were not collected into the accumulator, either the sets would be underestimated, or the *misses* would need to be corrected by a subsequent process, leading to longer reaction times. In this experiment, we tested whether such *miss* effects occur. Fifty participants judged numerosities of briefly presented sets of circles. In some conditions, circles were arranged such that some were inside others. This circle nesting was expected to increase the miss rate, since previous research had indicated that items in nested configurations cannot be preattentively individuated in parallel. Logically, items in a set that cannot be simultaneously individuated cannot be simultaneously added to an accumulator. Participants' response times were longer and their estimations were lower for sets whose configurations yielded greater levels of nesting. The level of nesting in a display influenced estimation independently of the total number of items present. This indicates that miss effects, predicted by the accumulator model, are indeed seen in ANS estimation. We speculate that ANS biases might, in turn, influence cognition and behavior, perhaps by influencing which kinds of sets are spontaneously counted.

Keywords Perceptual categorization and identification · Object-based attention · Reaction time methods

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Visual nesting impacts approximate number system estimation

We have long known that people can quickly— although imprecisely—assess numerosities of large sets without counting (Jensen, Reese, & Reese, 1950; Jevons, 1871; Kaufman, Lord, Reese, & Volkmann, 1949; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). The nature of this mechanism is a matter of debate (see Franconeri, Bemis, & Alvarez, 2009). It is of both theoretical and practical importance that we establish what factors may impact this estimation ability, particularly since individuals' skill at distinguishing numerosities without counting is linked to mathematical achievement (Feigenson, Dehaene, & Spelke, 2004; Halberda, Mazzocco, & Feigenson, 2008; National Mathematics Advisory Panel, 2008).

The mechanism that produces these estimates is known as the approximate number system (ANS). The ANS can process numerosities presented or produced both serially and simultaneously (Cordes, Gelman, Gallistel, & Whalen, 2001; Meck & Church, 1983; Revkin et al., 2008; Taves, 1941). ANS estimations can be influenced by perceptual factors, such as regularity of spacing (Ginsburg, 1976, 1978; Taves, 1941), perceived area (van Oeffelen & Vos, 1982; Vos, van Oeffelen, Tibosch, & Allik, 1988), and item segmentation (Franconeri et al., 2009). While some claim that ANS estimations are based solely on continuous extent features, such as area and density (Mix, Huttenlocher, & Levine, 2002), other research indicates that the ANS can assess numerosity even when continuous extent is controlled (Cordes & Brannon, 2008; Hurewitz, Gelman, & Schnitzer, 2006).

Meck and Church's (1983) *accumulator* model proposes that numerosities are assessed by tallying items, rather than derived from continuous extent. This model can be illustrated with the metaphor of a *graduated cylinder*. One "cup" is added to the "cylinder" for each item in the to-be-enumerated set, with the resulting "height" of the "liquid" in the "cylinder" indicating the set's numerosity (Cordes et al., 2001). Dehaene and Changeux (1993) offered a similarly item-based model capable of simultaneous and serial enumerations.

Chesney and Haladjian (2011) noted that the accuracy of such item-based models depends upon each item being tallied exactly once. If items were missed, such models would underestimate sets' numerosities. These misses offer one pathway by which perceptual variables may influence estimation. Logically, items cannot be tallied without first being individuated. Thus, if one uses an item-based system to enumerate sets that include items that cannot be simultaneously individuated, such items must either be tallied serially, which would take additional time, or be missed, which would tend to result in systematic underestimations of set numerosities. In the present experiment, we tested this prediction by having adults make numerical judgments across conditions that varied the likelihood that items would be missed. We manipulated this miss rate by arranging items such that some contained others. Such visual nesting is proposed to hinder simultaneous individuation[1] (Trick & Enns, 1997a, b; Trick & Pylyshyn, 1993, 1994). Participants viewed displays of circles in various fully nested, nonnested, and partially nested configurations at a rate intended to suppress verbal counting. Individuals were asked to indicate whether the cardinality of the circle set was greater than or less than a given digit value as quickly as possible without making too many mistakes. We predicted that as nesting increased, reaction times (RTs) would increase and/or numerosity would be underestimated.

Method

Participants

Fifty undergraduates (22 female) at Rutgers University, New Brunswick participated for course credit.

[1] Trick and her colleagues (Trick & Enns, 1997a, 1997b; Trick & Pylyshyn, 1993, 1994) concluded that people cannot preattentively individuate nested items because people cannot subitize (quickly and accurately enumerate without counting) nested items and people can only subitize items that they can preattentively individuate.

Design

Using forced choice discrimination tasks (see Kingdom & Prins, 2010), we had participants indicate whether the number of circles in a presented set (three to nine) was greater than the value of a subsequently presented digit ("5," "6," or "7"). When individuals perceive numerosities and digit values as equal, "circles more" response rates should approach .5, and RTs should peak. Circles were presented in one of three general configurations (described below) that varied the level of nesting present in the display: *nonnested*, *fully nested*, and *partially nested*. Hence, the experiment utilized a 7 (circle set size) × 3 (digit value) × 3 (configuration) within-subjects design.

Apparatus and stimuli

The experiment was programmed in MATLAB using Psychophysics Toolbox (Brainard, 1997) and was presented on an Apple® Power Mac G5 with a 17-in. flat screen LCD monitor (resolution, 1,280 × 1,024). Participants sat ~60 cm from the screen, yielding ~30° × ~25° of visual angle. No restraints were used to prevent head or eye movement.

Circles composed of 1-pixel thick black lines were displayed against a white background. Circle size was randomized, with the constraints that the average total perimeter of the circles (~3,200 pixels) be constant across set numerosity and topological arrangement and that the circle diameter be at least 24 pixels (0.62°). The mean circle diameter was ~168 pixels (~4.2°). Circles or, in some conditions, groups of nested circles were randomly placed in nonoverlapping positions. The smallest allowable separation between nested circles was 10 pixels (0.26°). Circles were restricted to appear in the central 880 × 824 pixel (22.68° × 21.24°) region of the screen, unless no nonoverlapping configuration within this central region could be found. In those cases (1.2 % of trials), the entire screen was used.

Nonnested

On nonnested trials, circles were arranged so as not to contain any others (see Fig. 1).

Fully nested

On fully nested trials, circles were arranged concentrically. Average spacing between adjacent circles was held constant across set numerosities. Sizes of individual circles in a set were matched to a randomly selected (without replacement) set of nonnested circles of the same numerosity (see Fig. 1).

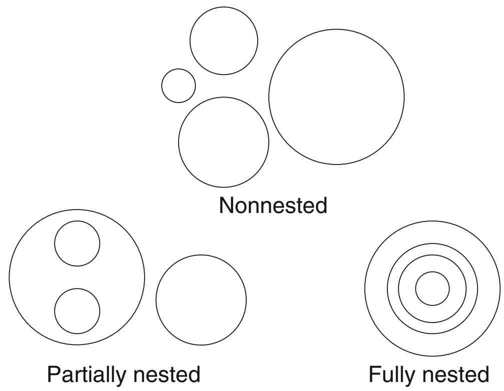

Fig. 1 Illustrations of nonnested, partially nested, and fully nested configurations composed of four circles

Partially nested

Partially nested stimuli were composed of various kinds of circle subgroups in specific configurations, henceforth known as *singles*, *doubles*, *triples*, and *quadruples* (see Fig. 2). A single was a single circle. A double was composed of a concentric pair. A triple was composed of an outer circle containing two equal smaller circles evenly spaced on the outer circle's horizontal or vertical diameter. A quadruple was composed of an outer circle containing three equal smaller circles arranged in an upward- or downward-pointing equilateral triangle around the outer circle's center. Orientation of inner circles in triples and quadruples was randomly determined. For each circle set cardinality, three topological arrangements were defined that specified the number of singles, doubles, triples, and quadruples to use in constructing the set (see Table 1). Partially nested trials' topological arrangements were randomly drawn from these possibilities. For circle sets of numerosities 6, 7, and 8, one of the arrangements had one nesting (one triple), and two of the arrangements had two nestings: one with two triples, and another with one double and one quadruple (see Table 1). The average distances between circles composing doubles and quadruples were designed to be smaller than those for triples. Total perimeter was matched to a randomly selected (without replacement) set of nonnested circles of the same numerosity (see Fig. 1).

Procedure

Before testing, participants were told that they would be shown sets of circles followed by a digit. They were instructed to hit the "F" key when the number of circles displayed was greater than the value of the digit and "J" for the reverse. Participants were warned that they would not have enough time to count the circles but should "just make the best guess [they could]." They were told to include all of the circles when determining numerosity, no matter where they appeared on the screen or whether they were contained by another circle. Participants were asked to answer as quickly as possible "without making too many mistakes," since they were being timed. Testing began after participants completed nine practice trials. To ensure understanding of the task, the experimenter monitored whether participants responded correctly on these practice trials and would clarify the instructions for those who made errors. An additional round of practice was allowed at the participant's request.

At the start of each trial, a set of three to nine circles in a nonnested, fully nested, or partially nested configuration was presented for 250 ms (a rate intended to suppress verbal counting; see Jensen et al., 1950; Kaufman et al., 1949; Revkin et al., 2008). This was followed by a 125-ms presentation of a plaid mask. Next, the digit "5," "6," or "7" appeared in the center of the screen. This digit remained until the participant responded. Responses were followed by a gray screen prompting participants to hit any key to continue. Participants could take a break at any time by waiting to press this key. Testing consisted of 1,134 trials, 18 for each cell in the 7 (circles) × 3 (digit) × 3 (configuration) design. Trials were presented in random order. No trial feedback was given. The computer recorded participants' responses and RTs. Testing took ~50 min per participant.

Results and discussion

Preliminary analyses indicated that a significant majority of the individual participants demonstrated the predicted effects.[2] Thus, we pooled the participants' data for the following analyses.

[2] For each participant, we calculated their nine best-fitting psychometric curves describing the proportion of "circles more" responses as a function of the circle set cardinality in the various configuration and digit conditions. Functions resolved in all but four cases (one in the fully nested condition where the digit was 7, one in the nonnested condition where the digit was 5, and two in the partially nested condition where the digit was 5). We compared individuals' PSEs and slopes found for nonnested and fully nested configurations with the same digit. Of the 48 individuals for whom all relevant curves resolved, 40 had higher PSEs for fully nested than for nonnested configurations in all three digit conditions (binomial test, $p < .0005$), and 37 had lower slopes for fully nested than for nonnested configurations in all three digit conditions (binomial test, $p < .0005$). Additionally, 45 of the 50 participants had longer mean RTs in the partially nested condition than in the nonnested condition (binomial test, $p < .0005$). A set of fifty 2 (configuration: nonnested vs. partially nested) × 3 (digit: 5, 6, and 7) × 7 (circles: three to nine) ANOVAs on individual participants' RTs demonstrated that this difference was significant for 29 participants and marginally significant for an additional 4.

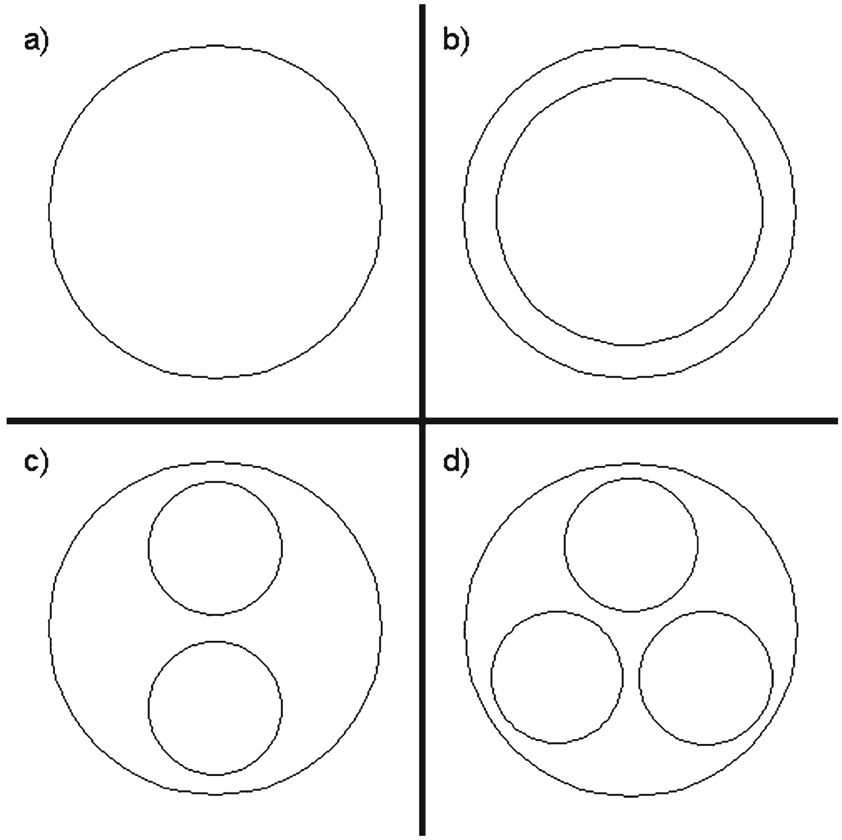

Fig. 2 Examples of kinds of circle subgroups used to construct stimuli in partially nested configurations: **a** single, **b** double, **c** triple, **d** quadruple

Table 1 Circle subgroups used to construct the predefined topological arrangements of circles for sets of each numerosity in the partially nested condition

# Circles	Topological arrangement				Likelihood of arrangement
	# Singles	# Doubles	# Triples	# Quadruples	
3	0	0	1	0	9.52 %
	1	1	0	0	4.76 %
4	1	0	1	0	4.76 %
	0	0	0	1	4.76 %
	2	1	0	0	4.76 %
5	2	0	1	0	4.76 %
	1	0	0	1	4.76 %
	0	1	1	0	4.76 %
6	3	0	1	0	4.76 %
	0	1	0	1	4.76 %
	0	0	2	0	4.76 %
7	4	0	1	0	4.76 %
	1	1	0	1	4.76 %
	1	0	2	0	4.76 %
8	5	0	1	0	4.76 %
	2	1	0	1	4.76 %
	2	0	2	0	4.76 %
9	6	0	1	0	4.76 %
	0	1	1	1	4.76 %
	0	0	3	0	4.76 %

Coding and analyses

ANS estimations are inherently variable (Cordes et al., 2001). Thus, participants were expected to be somewhat uncertain as to whether the numerosity of the set was greater than the value of the digit. Accordingly, “circles more” response rates should reflect S-shaped functions approaching .5 as the ratio of the perceived numerosity of the set to the digit’s value approaches 1 (the point of subjective equality, PSE) (see Hurewitz et al., 2006; Kingdom & Prins, 2010; Moyer & Landauer, 1967), This function’s slope indicates people’s ability to discriminate between circle sets’ numerosities; higher slopes indicate greater discriminability. Indeed, such curves were seen (see Fig. 3). We calculated the best-fitting cumulative normal curves for the proportion of “circles more” responses to a given circle set size for trials in the nine 3 (digit) × 3 (configuration) conditions. These are reported in Table 2.

As the ratio of the circle set’s perceived numerosity to the digit’s value approaches 1, comparing circle and digit numerosity should become more difficult (see Hurewitz et al., 2006; Moyer & Landauer, 1967). Thus, RTs should peak at the PSEs, with the rates of change indicating the circle sets’ discriminability. Results matched these predictions (see Fig. 3). These curves are well fit by quadratic functions over the range tested, with quadratic terms indicating discriminability and vertices indicating PSEs (see Table 3).

Effects of configuration

Biased estimations

As was predicted, nesting circles inside each other biased ANS estimations downward. Participants had lower numerosity estimates for circles in fully nested configurations than for nonnested sets (see Fig. 3 and Table 2). In the nonnested condition, “circles more” response rates approached .5 as the numerosity of the circles approached the digit’s value, indicating PSEs near the digits’ values. This indicates that participants were accurately assessing numerosities of nonnested sets. However, in the fully nested condition, “circles more” response rates did not approach .5 until the circles’

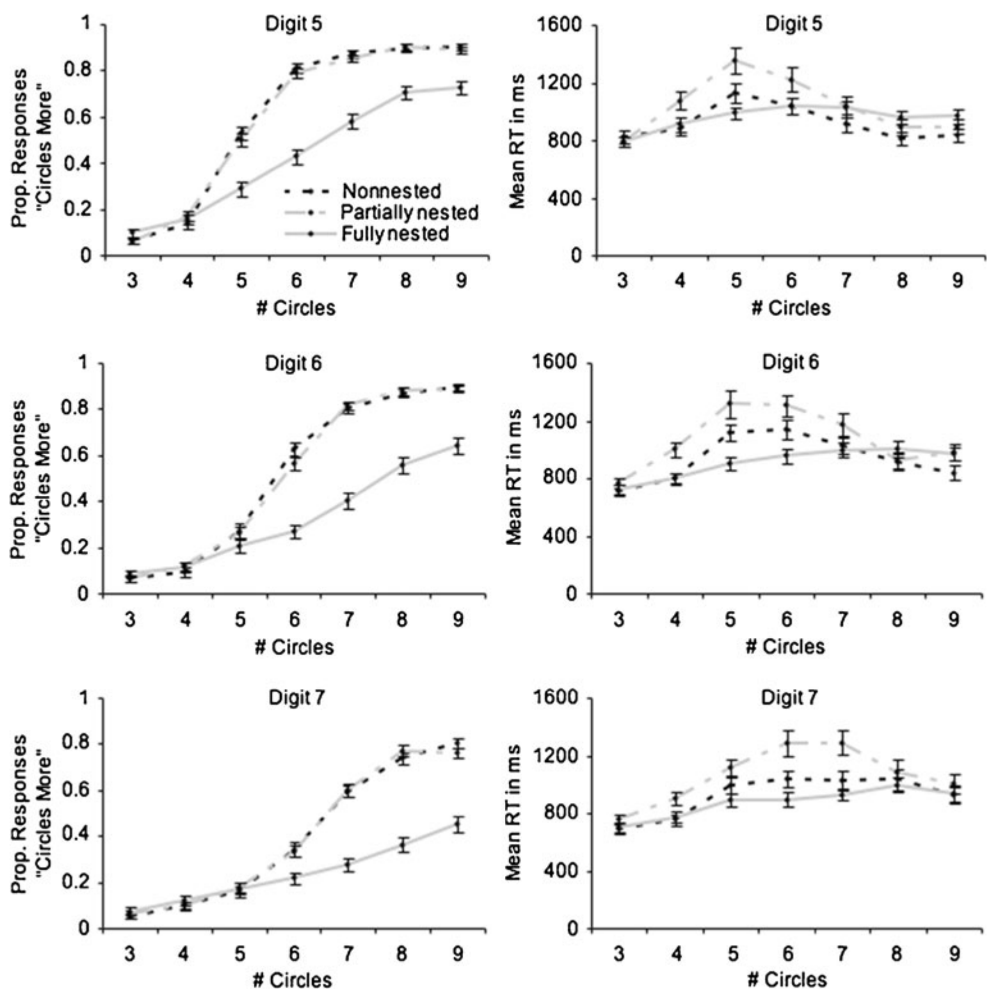

Fig. 3 Proportion of “circles more” responses (left column) and mean response times (RTs; right column) when the comparison digit was 5 (top row), 6 (middle row), or 7 (bottom row), as a function of the number of circles displayed and their configuration. Error bars represent standard errors. Analyses of individual participants’ performance were consistent with these findings

Table 2 Points of subjective equality (PSEs) and slopes for the best-fitting psychometric curves for the "circles more" response rates as a function of circle numerosity in the nonnested, fully nested, and partially nested configuration conditions, for each of the three digit conditions

Digit	Configuration	PSE (*SD*)	Slope (*SD*)	$R^{2\,a}$
5	Nonnested	5.0644 (.0300)	1.0801 (.0420)	.854
	Partially nested	5.1255 (.0335)	0.9198 (.0335)	.859
	Fully nested	6.6971 (.0529)	0.4081 (.0125)	.596**
6	Nonnested	5.7902 (.0344)	0.8339 (.0278)	.826
	Partially nested	5.8380 (.0352)	0.8052 (.0267)	.850
	Fully nested	7.7661 (.0650)	0.3859 (.0139)	.484**
7	Nonnested	6.8237 (.0415)	0.5908 (.0181)	.749
	Partially nested	6.8200 (.0429)	0.5618 (.0170)	.747
	Fully nested	9.4711 (.1514)	0.2747 (.0140)	.297**

Best-fitting curves and standard deviations were calculated using the Palamedes Toolbox (Prins & Kingdom, 2009), on the basis of cumulative normal distributions and 5,000 bootstrapping iterations. Upper and lower asymptotes were fixed at .06 and .94, respectively, on the basis of the 6 % error rate in the nonnested condition with three circles. Since three is in the subitizing range, one would predict that a perfectly attentive participant would be correct on all such trials. Therefore, we attribute these errors to a 12 % lapse rate, assuming that lapsed participants will guess correctly on half the trials

[a] R^2s refer to the proportions of explained variance of the individual participants' mean "circles more" response rates to the different circle numerosities for the indicated configuration and digit conditions

**$p < .0005$: significance of differences in response rates between nonnested and fully nested or between nonnested and partially nested circle sets for a given comparison digit established by a 2 (configuration) × 7 (circle numerosity) repeated measures ANOVA (see Appendix 1), as indicated by either a main effect of configuration or an interaction between configuration and circle numerosity, whichever is greater

numerosities exceeded the digit's value by ~2, indicating that the participants were underestimating numerosities of the fully nested sets. Also as predicted, participants were less able to discriminate numerosities in the fully nested than in the nonnested condition, as indicated by the shallower slopes of the best-fitting psychometric curves (see Fig. 3 and Table 2).

These findings cannot be fully explained by the influence of continuous extent on participants' numerosity judgments. Different arrangements of the same circles were used to construct the fully nested and nonnested stimuli and, thus, were matched on both perimeter and total circle area. Furthermore, a logistic regression demonstrated configuration condition (nonnested vs. fully nested) influenced "circles more" response rates [$B = 3.634$, $z = 30.5$, $Wald(1, N = 37800) = 935.9$, $p < .0005$] even when controlling for digit, circle numerosity, and unique circle area.[3]

RT data further support the conclusion that participants underestimated numerosities in the fully nested condition (see Fig. 3 and Table 3). RTs should peak at the PSE, since it is there that numerosity discrimination is most difficult (see Hurewitz et al., 2006; Moyer & Landauer, 1967). RTs in the fully nested condition tended to peak when the circles' numerosities were 1 or 2 greater than the digit's value, as indicated by the vertex of the best-fitting quadratic function. This indicates that participants underestimated the circles' numerosities. Furthermore, RTs were less affected by circle numerosity in the fully nested than in the nonnested condition, as indicated by the quadratic term of the best-fitting quadratic function. Since there was no mean difference in RTs between the nonnested (M = 924 ms, SE = 43 ms) and fully nested (M = 911 ms, SE = 38 ms) conditions [2 (configuration: nonnested vs. fully nested) × 3 (digit) × 7 (circles) repeated measures ANOVA; main effect of configuration: $F(1, 49) = 0.6$, $p = .454$, $\eta_p^2 = .011$], these different rates of change indicate that participants were less able to discriminate numerosities in fully nested condition.

Longer RTs

Recall that we predicted that nesting could lead to longer RTs if participants corrected their initial ANS estimates in a subsequent process. Such longer RTs were seen in the partially nested condition. Although participants did not differ in PSEs or set size discriminability between the nonnested and partially nested conditions (see Fig. 3 and Table 2), RTs were longer in the partially nested (M = 1,054 ms, SE = 56 ms) than in the nonnested (M = 924 ms, SE = 43 ms) condition [2 (configuration: nonnested vs. partially nested) × 3 (digit) × 7 (circles) repeated measures ANOVA; main effect of configuration: $F(1, 49) = 6.8$, $p = .012$, $\eta_p^2 = .112$; see Fig. 3, and Table 3]. Indeed, several participants spontaneously indicated that they "knew there were more circles than [they] thought" when nesting was present and, thus, shifted their numerosity estimates upward. One observer (not in the

[3] Unique circle area refers to the total circle area in the nonnested trials and the area of the outer circle in the fully nested trials.

Table 3 Best-fitting quadratic curves for the participants' mean response times as a function of circle numerosity in the nonnested, fully nested, and partially nested configuration conditions, for each of the three digit conditions

Digit	Configuration	Quadratic term (*SE*)	Linear term (*SE*)	*y*-intercept (*SE*)	Vertex	$R^{2\ a}$
5	Nonnested	$-.024^{++}$ (.006)	$.273^{++}$ (.067)	.224 (.188)	5.69	.053
	Partially nested	$-.044^{++}$ (.007)	$.510^{++}$ (.088)	−.281 (.246)	5.79	.096**
	Fully nested	$-.017^{+}$ (.005)	$.225^{++}$ (.063)	.275 (.175)	6.62	.048**
6	Nonnested	$-.038^{++}$ (.006)	$.478^{++}$ (.069)	$-.402^{+}$(.192)	6.29	.124
	Partially nested	$-.047^{++}$ (.007)	$.574^{++}$ (.089)	$-.504^{+}$(.250)	6.11	.106*
	Fully nested	$-.012^{+}$ (.005)	$.193^{+}$ (.063)	.243 (.175)	8.04	.081**
7	Nonnested	$-.026^{++}$ (.006)	$.358^{++}$ (.072)	−.191 (.201)	6.88	.103
	Partially nested	$-.042^{++}$(.008)	$.555^{++}$ (.093)	$-.574^{+}$(.260)	6.61	.110*
	Fully nested	$-.010^{+}$ (.005)	$.164^{+}$ (.059)	.295 (.163)	8.20	.079*

$^{+}$ $p < .05$

$^{++}$ $p < .0005$

[a] R^2 s refer to the proportions of explained variance of the individual participants' mean reaction times to the different circle numerosities for the indicated configuration and digit conditions

*$p < .05$, **$p < .0005$: significance of differences in response times between nonnested and fully nested or between nonnested and partially nested circle sets for a given comparison digit established by a 2 (configuration) × 7 (circle numerosity) repeated measures ANOVA (see Appendix 2), as indicated by either a main effect of configuration or an interaction between configuration and circle numerosity, whichever is greater

original sample) specifically claimed to have used a multistep process to enumerate circles in a partially nested configuration, saying "I saw there was a two and a three and I added them." It seems that at least some participants detected that nested subgroups were present[4] and adjusted their numerosity estimates upward.[5]

A closer examination of different topological arrangements of partially nested circle sets of numerosities 6, 7, and 8 lends further support to the hypothesis that these longer RTs resulted from participants correcting initial biases. In these conditions, one arrangement had one nesting (one triple), and two arrangements had two nestings (two triples or one double and one quadruple; see Table 1 and Fig. 4). This allowed different levels of nesting to be compared within the partially nested condition, while accounting for different forms of nesting and spacing between circles. Participants gave "circles more" responses at higher rates for the one-nesting arrangement ($M = .766$, $SE = .016$) than for either of the two-nesting arrangements (two triples, $M = .701$, $SE = .014$; double and quadruple, $M = .706$, $SE = .016$),[6] but there was no significant difference between the two-nesting arrangements [respective paired samples *t*-tests: $t(49) = 4.69$, $p < .0005$; $t(46) = 4.14$, $p < .0005$; $t(46) = 0.06$, $p > .95$] (see Footnote 6). This indicates that participants produced lower numerosity estimates in the two-nesting arrangements specifically due to the level of nesting, rather than perceived area or subgroup configuration.

Additionally, RTs increased with nesting. Participants were faster for the one-nesting arrangement (M = 1,057 ms, SE = 69 ms) than for either of the two-nesting arrangements (two triples, M = 1,189 ms, SE = 71 ms; double and quadruple, M = 1,191 ms, SE = 77 ms) (see Footnote 6), but there was no significant difference between the two-nesting arrangements [respective paired samples *t*-tests: $t(49) = 3.81$, $p < .0005$; $t(46) = 3.98$, $p < .0005$; $t(46) = 0.62$, $p > .5$) (see Footnote 6). This suggests that the time needed to correct ANS estimates increased with the level of nesting.

Conclusions, caveats, and considerations

These results match the predicted performance of an item-based system enumerating simultaneously presented items under conditions where impaired individuation increases miss rates. In the partially nested condition, the number of

[4] Subgroups contained circles arranged in canonical patterns that should render them easily detectable. Indeed, it has previously been proposed that recognition of such canonical patterns is beneficial to enumeration (Mandler & Shebo, 1982; Wolters, Vankempen, & Wijlhuizen, 1987).

[5] One method that would allow people to subsequently correct estimations once items are out of view would be to separately accumulate the different kinds of subgroups when the stimuli are initially presented. People can simultaneously maintain multiple enumerations for different kinds of items (Feigenson, 2008). If one treats different subgroups as different kinds, an individual who has accumulated two singles and two triples can subsequently derive that there are eight circles: 2 (the accumulated singles) + 3 + 3 (three for each of the accumulated triples).

[6] Three participants had at least one missing cell for two-nesting cases using one double and one quadruple. These participants' data were excluded from the relevant analyses.

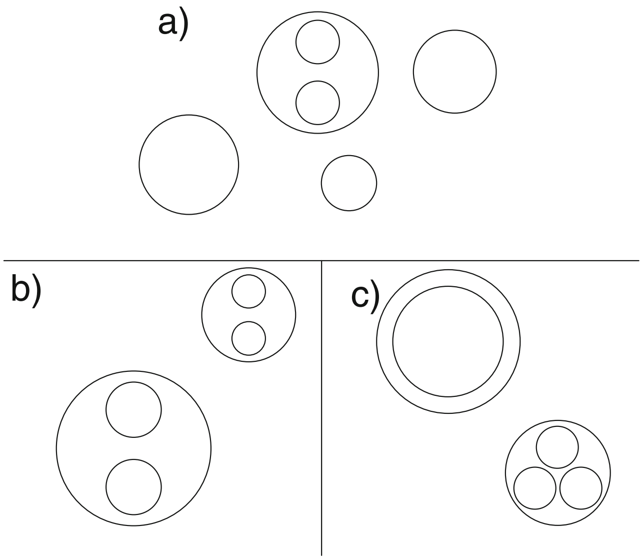

Fig. 4 Six circles in partially nested arrangements with **a** one nesting from one triple, **b** two nestings from two triples, and **c** two nestings from one double and one quadruple

nestings was independent of the number of circles, and thus, resulting biases could be successfully corrected, perhaps by adjusting initial estimations upward for each detected subgroup (see Footnote 5). This resulted in accurate estimations but longer RTs. Such a correction method was not available in the fully nested condition, since there was always only one "subgroup" of concentric circles. Thus, numerosities were underestimated. Further studies are needed to determine at which level of processing correction occurs.

We further speculate that by biasing people's automatic ANS estimations (see Hurewitz et al., 2006), perceptual factors such as object nesting can influence people's intuitions about the numerosity of visually presented stimuli and, ultimately, what sets people decide to count. Miss effects on enumeration would explain Franconeri et al.'s (2009) finding that figures containing items connected by strong lines were perceived as less numerous than figures without such connections; these lines may have caused people to enumerate pairs, rather than individual items.[7]

[7] Franconeri et al. (2009) asked participants to compare perceived numerosities rather than make judgments regarding specific numerosities, and they did not report RTs. Thus, they did not investigate the effects of impaired individuation on ANS estimation in the manner we have done in the present experiment. However, we find it telling that Franconeri et al.'s participants judged sets to be more numerous when items were connected (and thus were more difficult to individuate) than when they were not connected, just as our participants produced lower numerosity estimates of sets when circles were fully nested (and thus, according to Trick and her colleagues, were more difficult to individuate; Trick & Enns, 1997a, 1997b; Trick & Pylyshyn, 1993, 1994) than when they were nonnested. This therefore serves as convergent evidence that object nesting was indeed impairing individuation, as was our intent.

Fig. 5 An illustration of six plates, arranged in two stacks of three, such that they are visually nested

Furthermore, our participants were clearly instructed to enumerate all of the circles, regardless of any nesting relationships present. However, in many real-world contexts, people must determine for themselves what the most appropriate countable set may be. Consider Fig. 5. This image could be described as six plates (counting the plates) or as two stacks of plates (counting the stacks). This nesting of plates does not merely construct the stacks; it makes it more difficult to enumerate all the plates than if they were not nested.

Although many sets may be present in any given scene, people may tend to count those sets that are most easily enumerated by the ANS; when ANS estimation is easier, one's intuitions about set numerosity would be more likely to resemble the true count. This might explain previous findings (Chesney, 2009) that people were less likely to count illustrated items that contained other items, such as a window through which one could see items "outside." Those items were visually nested and, thus, would have been more difficult for the ANS to enumerate.

This possibility is particularly interesting, given Halberda et al.'s (2008) finding that one's ability to use the ANS to distinguish between numerosities correlates with mathematical proficiency. It is plausible that individual differences in sensitivity to perceptual variables that influence ANS estimation also influence the development of mathematical proficiency. These findings highlight the importance of being conscious of how the perceptual features of stimuli may bias enumerations, particularly when gauging individuals' ability to distinguish numerosity.

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Appendix 1

Table 4 Results of a series of six 2 (configuration) × 7 (circle numerosity) repeated measures ANOVAs comparing participants' "circles more" response rates in the nonnested and fully nested configuration conditions and in the nonnested and partially nested configuration conditions for each of the three digit conditions

ANOVA: configuration and digit conditions	Main effect of configuration	Main effect of circle numerosity	Configuration × circle numerosity interaction
Non- vs. fully nested, digit 5	$F(1, 49) = 63.5$	$F(2, 294) = 466.9$	$F(2, 294) = 36.7$
	$p < .0005$, $\eta_p^2 = .564$	$p < .0005$, $\eta_p^2 = .905$	$p < .0005$, $\eta_p^2 = .428$
Non- vs. partially nested, digit 5	$F(1, 49) = 0.6$	$F(2, 294) = 639.4$	$F(2, 294) = 1.1$
	$p = .451$, $\eta_p^2 = .012$	$p < .0005$, $\eta_p^2 = .929$	$p = .373$, $\eta_p^2 = .022$
Non- vs. fully nested, digit 6	$F(1, 49) = 70.1$	$F(2, 294) = 378.2$	$F(2, 294) = 41.7$
	$p < .0005$, $\eta_p^2 = .589$	$p < .0005$, $\eta_p^2 = .885$	$p < .0005$, $\eta_p^2 = .460$
Non- vs. partially nested, digit 6	$F(1, 49) = 0.2$	$F(2, 294) = 519.6$	$F(2, 294) = 1.57$
	$p = .641$, $\eta_p^2 = .004$	$p < .0005$, $\eta_p^2 = .914$	$p = .156$, $\eta_p^2 = .031$
Non- vs. fully nested, digit 7	$F(1, 49) = 55.9$	$F(2, 294) = 249.9$	$F(2, 294) = 51.4$
	$p < .0005$, $\eta_p^2 = .533$	$p < .0005$, $\eta_p^2 = .831$	$p < .0005$, $\eta_p^2 = .512$
Non- vs. partially nested, digit 7	$F(1, 49) = 0.0$	$F(2, 294) = 347.4$	$F(2, 294) = 1.1$
	$p = .880$, $\eta_p^2 = .000$	$p < .0005$, $\eta_p^2 = .876$	$p = .391$, $\eta_p^2 = .021$

Appendix 2

Table 5 Results of a series of six 2 (configuration) × 7 (circle numerosity) repeated measures ANOVAs comparing participants' response times in the nonnested and fully nested configuration conditions and in the nonnested and partially nested configuration conditions for each of the three digit conditions

ANOVA: configuration and digit conditions	Main effect of configuration	Main effect of circle numerosity	Configuration × circle numerosity interaction
Non- vs. fully nested, digit 5	$F(1, 49) = 3.3$	$F(2, 294) = 18.4$	$F(2, 294) = 7.7$
	$p = .077$, $\eta_p^2 = .063$	$p < .0005$, $\eta_p^2 = .273$	$p < .0005$, $\eta_p^2 = .136$
Non- vs. partially nested, digit 5	$F(1, 49) = 5.5$	$F(2, 294) = 42.0$	$F(2, 294) = 4.3$
	$p = .024$, $\eta_p^2 = .100$	$p < .0005$, $\eta_p^2 = .461$	$p < .0005$, $\eta_p^2 = .080$
Non- vs. fully nested, digit 6	$F(1, 49) = 2.2$	$F(2, 294) = 29.2$	$F(2, 294) = 12.0$
	$p = .147$, $\eta_p^2 = .043$	$p < .0005$, $\eta_p^2 = .374$	$p < .0005$, $\eta_p^2 = .197$
Non- vs. partially nested, digit 6	$F(1, 49) = 6.1$	$F(2, 294) = 46.5$	$F(2, 294) = 2.5$
	$p = .017$, $\eta_p^2 = .110$	$p < .0005$, $\eta_p^2 = .487$	$p = .023$, $\eta_p^2 = .048$
Non- vs. fully nested, digit 7	$F(1, 49) = 4.0$	$F(2, 294) = 30.0$	$F(2, 294) = 3.1$
	$p = .050$, $\eta_p^2 = .076$	$p < .0005$, $\eta_p^2 = .380$	$p = .005$, $\eta_p^2 = .060$
Non- vs. partially nested, digit 7	$F(1, 49) = 8.1$	$F(2, 294) = 34.2$	$F(2, 294) = 3.5$
	$p = .007$, $\eta_p^2 = .141$	$p < .0005$, $\eta_p^2 = .411$	$p = .002$, $\eta_p^2 = .067$

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