Language and the Origin of Numerical Concepts

Rochel Gelman* and C. R. Gallistel*

Reports of research with the Pirahã and Mundurukú Amazonian Indians of Brazil lend themselves to discussions of the role of language in the origin of numerical concepts. The research findings indicate that, whether or not humans have an extensive counting list, they share with nonverbal animals a language-independent representation of number, with limited, scale-invariant precision. What causal role, then, does knowledge of the language of counting serve? We consider the strong Whorfian proposal, that of linguistic determinism; the weak Whorfian hypothesis, that language influences how we think; and that the "language of thought" maps to spoken language or symbol systems.

Intuitively, our thoughts are inseparable from the words in which we express them. This intuition underlies the strong form of the Whorfian hypothesis, namely, that language determines thought (aka "linguistic determinism"). Many cognitive scientists find the strong hypothesis unintelligible and/or indefensible (*1*), but weaker versions of it, in which language influences how we think, have many contemporary proponents (*2*, *3*).

The strong version rules out the possibility of thought in animals and humans who lack language, although there is an abundant experimental literature demonstrating quantitative inference about space, time, and number in preverbal humans (*4*), in individuals with language impairments (*5*), and in rats, pigeons, and insects (*6*). Another problem is the lack of specific suggestions as to how exposure to language could generate the necessary representational apparatus. It would be wonderful if computers could be made to understand the world the way we do just by talking to them, but no one has been able to program them to do this. This failure highlights what is missing from the strong form of the hypothesis, namely, suggestions as to how words could make concepts take form out of nothing.

The antithesis of the strong Whorfian hypothesis is that thought is mediated by language-independent symbolic systems, often called the language(s) of thought (*7*). Under this account, when humans learn a language, they learn to express in it concepts already present in their prelinguistic system(s). Their prelinguistic systems for representing the world are language-like only in that they are compositional: Larger, more complex meanings (concepts) are built up by the combination of elementary meanings.

Psychology and Cognitive Science, Rutgers University, 152 Frelinghuysen Road, Piscataway, NJ 08854. USA.

*To whom correspondence should be addressed. E-mail: rgelman@ruccs.rutgers.edu (R.G.); Galliste@ruccs.rutgers.edu (C.R.G.)

Recently reported experimental studies (*8*, *9*) with innumerate Pirahã and Mundurukú Indian subjects from the Brazilian Amazonia give evidence regarding the role of language in the development of numerical reasoning. Either the subjects in these reports have no true number words (*8*, *10*) or they have consistent, unambiguous words for one and two and more loosely used words for three and four (*9*). Moreover, they do not overtly count, either with number words or by means of tallies. Yet, when tested on a variety of numerical tasks—naming the number of items in a stimulus set, constructing sets of equivalent number, judging which of two sets is more numerous, and mental addition and subtraction—these subjects gave results indicative of an imprecise nonverbal representation of number, with a constant level of imprecision, measured by the Weber fraction. The Weber fraction for these subjects is roughly comparable to that of numerate subjects when they do not rely on verbal counting. In one of the reports, the stimulus sets had as many as 80 items, so the approximate representation of number in these subjects extends to large numbers.

Among the most important results in these reports are those showing simple arithmetic reasoning—mental addition, subtraction, and ordering. These findings strengthen the evidence that humans share with nonverbal animals a language-independent representation of number, with limited, scale-invariant precision, which supports simple arithmetic computation and which plays an important role in elementary human numerical reasoning, whether verbalized or not (*5*, *11*–*13*). Contrary to (*8*) and to reports in the secondary media, the results do not support the strong Whorfian view that a concept of number is dependent on natural language for its development. Indeed, they are evidence against it. The results are, however, consistent with the hypothesis that learning to represent numbers by some communicable notation (number words, tally marks, numerals) might facilitate the routine recognition of exact numerical equality.

These reports suggest that people with extremely limited or no verbal counting have the same nonverbal representation of number as do subjects with a fluent, well-developed verbal counting system. The long-established and robust symbolic size and distance effect is a principal line of evidence for this representation and for its importance in discussions of verbal numerical reasoning: Numerate subjects judge the ordering of symbolized number with ease, but they have no insight into how they do so. Most are surprised to learn that it takes them longer to decide that $3 > 2$ than it does to decide that $5 > 2$, whether the questions are posed symbolically (3 >? 5) or with arrays instantiating the numbers (Fig. 1, A and B). The reaction time for judgments of numerical order is a function of the ratio between the numbers being judged. The function is the same in monkeys as in numerate adults (Fig. 1, C and D).

The symbolic size and distance effects are generally taken to indicate that the determination of numerical order by the brain depends on imprecise mental magnitudes. These are hypothesized variables in the brain that vary systematically with number (and other quantitative dimensions of experience) and that form the basis for the subjective sense of magnitude. They are called mental or subjective magnitudes to distinguish them from the objective magnitudes that they represent. The mental magnitudes for repeated instantiations of an objective magnitude vary, forming what communications engineers call a signal distribution. The wider these distributions, the more imprecise are the representations. The extent to which two signal distributions overlap determines the likelihood of confusion about which distribution a signal belongs to, that is, which objective magnitude generated it. It is assumed that the extent of overlap between two signal distributions is determined by the ratio between the corresponding objective magnitudes (Fig. 2). The greater the likelihood of confusion, the more processing time is required to determine the proper distribution. This is the generally accepted explanation for the symbolic size and distance effects. It ties basic arithmetic reasoning (order judgments) with numerical symbols to an imprecise nonverbal representation of number.

On the non-Whorfian account, the mental magnitudes that represent number are an example of elementary nonlinguistic representations (meanings) for which numerate subjects have learned words. Subjects believe that the property denoted by "three" may be added to the property denoted by "two" to obtain the property denoted by "five" because this is already true for the prelinguistic concepts to which the words refer and from which they derive their meaning. That is, the mental magnitude that represents three may be mentally added to the mental magnitude that represents two to get the mental magnitude that represents five. Plausibly, the language learner comes to believe that those words have those meanings, precisely because she observes that their use is consistent with those meanings. Children hear, "Three and two are five" but not "Cow and big are red." From the syntactic frames in which words occur, much may be inferred about their referents (*14*, *15*).

In showing that subjects with no verbal counting system have a concept of approximate numerical magnitude like that of numerate subjects, these reports support the non-Whorfian view for the origins of our concept of number. However, there is more to the story. Numerate subjects have a strong intuition of exact numerical equality. Two plus two is exactly four, not roughly four, and the square root of two is not exactly equal to the proportion between any two count numbers, that is, to any rational number, although a rational number that is as close as one wishes may readily be found. This aspect of the meaning of number words is not readily explained by the assumption that it is the reference to imprecise mental magnitudes that gives number words their meanings. When the non-numerate subjects in these reports matched a set of four items to a set of five, or judged that 6 – 3 = 2, they gave evidence of being indifferent to exact numerical equality, an indifference not seen in numerate control subjects. Thus, the reports suggest that the learning of number words either creates a concept of exact numerical equality (a strong Whorfian hypothesis), or mediates the expansion of such a concept (a weaker Whorfian hypothesis), or directs attention to such a concept (a non-Whorfian hypothesis).

A current hypothesis of the second (weak Whorfian) kind is the two-systems hypothesis, which is that, in addition to the approximate representation of numerical magnitude, there is a second prelinguistic representation, limited to numbers from one to four (*16*). Because this second, small-number-only system is discrete and precise, exact equality is intrinsic to the representation. It comes from the identity of the representing symbols (e.g., II ≡ II). On these accounts, the acquisition of a verbal counting system mediates the extension of the notion of exact equality to our concept of numbers greater than four.

The reports from experiments with nonnumerate subjects do not offer much support for two-system hypotheses, because the innumerate subjects represent three and four imprecisely—and, arguably, even one and two. This finding is also problematic for closely related and long-popular hypotheses that postulate perceptlike representations of the numbers one to four (*17*, *18*). If these subjects have precise or percept-like prelinguistic representations of three and four, then, curiously, they have no words that refer unambiguously to them.

There are at least two conceptual problems with dual-representation hypotheses. First, if the words for the numbers one to four derive their meaning from discrete, noise-free prelinguistic symbols or percepts, then why are the symbolic size and distance effects seen in this range? (Recall that these effects are assumed to derive from the imprecision of the mental magnitude representation of number.) Second, the compositionality of number concepts is a sine qua non. If the brain represents three and seven in fundamentally different ways, how can it compose them arithmetically (order them, add them, etc.)? What representational form do the resulting hybrids have? This is particularly puzzling when two numbers beyond the discrete and precise range are subtracted to yield a number inside it, as in 7 – 5 = 2.

Nonetheless, reports of subjects who appear indifferent to exact numerical equality even for small numbers, and who also do not count verbally, add weight to the idea that learning a communicable number notation with exact nu-

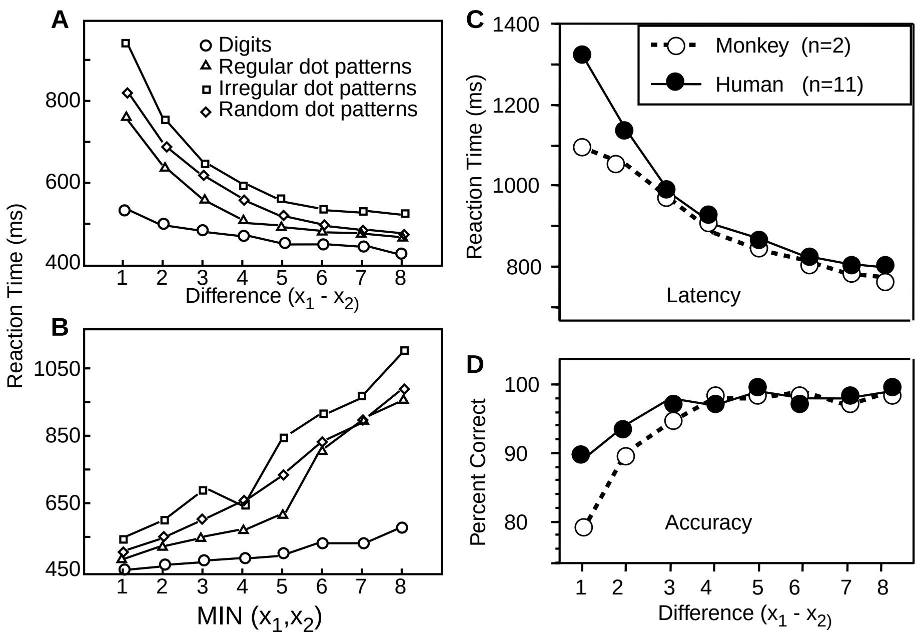

Fig. 1. The symbolic and nonsymbolic size and distance effects in the judgment of numerical order. (**A**) Time taken to make the order judgment as a function of the difference between two single digits (open circles) or between instantiations of the two numerosities (other symbols, nonsymbolic). (**B**) Reaction time as a function of the size of the smaller digit or number of stimulus items. (**C** and **D**) The distance effect for instantiated numbers is the same in humans and monkeys. [(A) and (B) are based on figures 1 and 2 in (*19*); (C) and (D) are based on figure 26.5 in (*12*).]

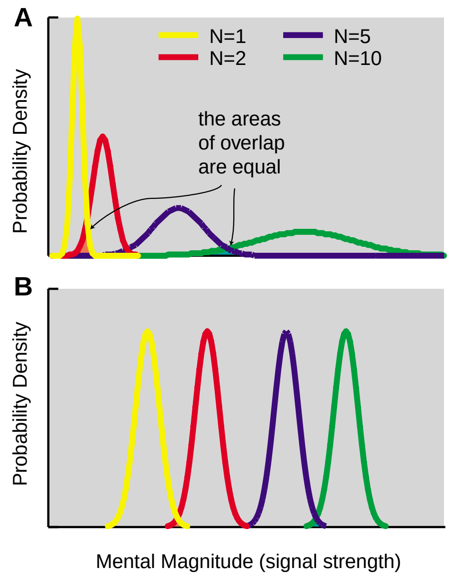

Fig. 2. Two common explanations for the size and distance effects. (**A**) Scalar variability. The mean mental magnitude is proportional to the number, as is the variability about this mean. Thus, the distributions are scale invariant, which means that the overlap between any two of them is determined by the ratio of their means. (**B**) Logarithmic compression. The mean mental magnitude is proportional to the logarithm of the number, whereas the variability is independent of it. Again, distributions for objective magnitudes that differ by a given ratio (e.g., 2:1) show the same overlap and, hence, the same potential for confusion about which distribution a particular signal properly belongs to.

merical reference may play a role in the emergence of a fully formed conception of number. The challenge now is to delineate that role.

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REVIEW

The Role of the Medial Frontal Cortex in Cognitive Control

K. Richard Ridderinkhof,[1,2]* Markus Ullsperger,[3] Eveline A. Crone,[4] Sander Nieuwenhuis[5]

Adaptive goal-directed behavior involves monitoring of ongoing actions and performance outcomes, and subsequent adjustments of behavior and learning. We evaluate new findings in cognitive neuroscience concerning cortical interactions that subserve the recruitment and implementation of such cognitive control. A review of primate and human studies, along with a meta-analysis of the human functional neuroimaging literature, suggest that the detection of unfavorable outcomes, response errors, response conflict, and decision uncertainty elicits largely overlapping clusters of activation foci in an extensive part of the posterior medial frontal cortex (pMFC). A direct link is delineated between activity in this area and subsequent adjustments in performance. Emerging evidence points to functional interactions between the pMFC and the lateral prefrontal cortex (LPFC), so that monitoring-related pMFC activity serves as a signal that engages regulatory processes in the LPFC to implement performance adjustments.

Flexible goal-directed behavior requires an adaptive cognitive control system for selecting contextually relevant information and for organizing and optimizing information processing. Such adaptive control is effortful, and therefore it may not be efficient to maintain high levels of control at all times. Here we review recent studies in cognitive neuroscience that have advanced our understanding of how the brain determines and communicates the need to recruit cognitive control. Convergent evidence suggests that the posterior medial frontal cortex (pMFC) and lateral prefrontal cortex (LPFC) are important contributors to cognitive control. Our focus is on the role of the pMFC in performance monitoring, especially in situations in which pMFC activity is followed by performance adjustments. Evaluating the adequacy and success of performance is instrumental in determining and implementing appropriate behavioral adjustments. For instance, detection of a performance error may be used to shift performance strategy to a more conservative speed/accuracy balance. Based on the evidence reviewed below, we develop the tentative hypothesis that one unified function of the pMFC is performance monitoring in relation to anticipated rewards. The monitored signals may index the failure (errors or negative feedback) or reduced probability (conflicts or decision uncertainty) of obtaining such rewards, and as such signal the need for increased control.

[1]Department of Psychology, University of Amsterdam, Roetersstraat 15, 1018 WB Amsterdam, Netherlands. [2]Department of Psychology, Leiden University, Wassenaarseweg 52, 2333 AK Leiden, Netherlands. [3]Max Planck Institute for Human Cognitive and Brain Sciences, Stephanstrasse 1A, 04103 Leipzig, Germany. [4]Center for Mind and Brain, University of California Davis, 202 Cousteau Place, Suite 201, Davis, CA 95616, USA. [5]Department of Cognitive Psychology, Vrije Universiteit, Van der Boechorststraat 1, 1081 BT Amsterdam, Netherlands.

*To whom correspondence should be addressed. E-mail: K.R.Ridderinkhof@uva.nl

Performance Monitoring

Flexible adjustments of behavior and reward-based association learning require the continuous assessment of ongoing actions and the outcomes of these actions. The ability to monitor and compare actual performance with internal goals and standards is critical for optimizing behavior. We first review evidence from primate, electrophysiological, and functional neuroimaging studies that points toward the importance of pMFC areas (Fig. 1A) in monitoring unfavorable performance outcomes, response errors, and response conflicts, respectively. These conditions have in common that they signal that goals may not be achieved or rewards may not be obtained unless the level of cognitive control is subsequently increased.

Although the pMFC can also be activated by positive events (such as rewards) (*1*, *2*), we focus here on negative events and their consequences. Because errors and conflicts are intrinsically negative, and because unfavorable outcomes are typically more consequential for the regulation of cognitive control than are favorable outcomes, our review focuses on the role of the pMFC in monitoring negative events.

Monitoring unfavorable outcomes. Electrophysiological recordings in nonhuman primates implicate the pMFC in monitoring performance outcomes. Distinct neuron populations in the pMFC, particularly in the supplementary eye fields and the rostral cingulate motor area (CMAr), are sensitive to reward expectancy and reward delivery (*1*, *3*, *4*). In addition, CMAr neurons exhibit sensitivity to unexpected reductions in reward (*5*). Likewise, specific groups of neurons in the depth of the cingulate sulcus (area 24c) react to response errors and to unexpected omissions of rewards (*5*). These findings are consistent with a role for these neuronal populations in comparing expected and actual outcomes.