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Naive Mathematics

Whether or not schooling is offered, children and adults all over the world develop an intuitive, naive mathematics. As long as number-relevant examples are part of their culture, people will learn to reason about and solve addition and subtraction problems with positive natural numbers. They also will rank order and compare continuous amounts, if they do not have to measure with equal units. The notion of equal units is hard, save for the cases of money and time. Universally, and without formal instruction, everyone can use money. Examples abound of child candy sellers, taxicab drivers, fishermen, carpenters, and so on developing fluent quantitative scripts, including one for proportional reasoning. Of note is that almost always these strategies use the natural numbers and nonformal notions of mathematical operations. For example, the favored proportions strategy for Brazilian fishermen can be dubbed the "integer proportional reasoning": the rule for reasoning is that one whole number goes into another *X* number of times and there is no remainder.

Intuitive mathematics serves a wide range of everyday math tasks. For example, Liberian tailors who have no schooling can solve arithmetic problems by laying out and counting familiar objects, such as buttons. Taxicab drivers and child fruit vendors in Brazil invent solutions that serve them well (Nunes, Schliemann, and Carraher 1993).

Two kinds of theories vie for an account of the origins and acquisition of intuitive arithmetic. One idea is that knowledge of the counting numbers and their use in arithmetic tasks builds from a set of reinforced bits of learning about situated counting number routines. Given enough learning opportunities, principles of counting and arithmetic are induced (Fuson 1988). Despite the clear evidence that there are pockets of early mathematical competence, young children are far from perfect on tasks they can negotiate. Additionally, the range of set sizes and tasks they can deal with is limited. These facts constitute the empirical foundation for the "bit-bit" theory and would seem to constitute a problem for the "principle-first" account of intuitive mathematics, which proposes an innate, domain-specific, learning-enabling structure. Although skeleton-like to start, such a structure serves to draw the beginning learner's attention to seek out, attend to, and assimilate number-relevant data—be these in the physical, social, cultural and mental environments—that are available for the epigenesis of number-specific knowledge.

True, there are many arithmetic reasoning tasks that young children cannot do, and early performances are shaky. But this would be expected for any learning account. Those who favor the principle-first account (Geary 1996; Gelman and Williams 1997) point to an ever-increasing number of converging lines of evidence: animals and infants respond to the numerical value of displays (Gallistel and Gelman 1992; Wynn 1995); retarded children have considerable difficulty with simple arithmetic facts, money, time, and novel counting or arithmetic tasks—despite extensive in-school practice (e.g., Gelman and Cohen 1988); preschool children distinguish between novel count sequences that are wrong and those which are unusual but correct; they also invent counting solutions to solve arithmetic problems (Siegler and Shrager 1984; Starkey and Gelman 1982); and elementary school children invent counting solutions to solve school arithmetic tasks in ways that differ from those they are taught in school (Resnick 1989). Moreover, there is cross-language variability in the transparency of the base rules for number word generation. For example, in Chinese, the words for 10, 11, 12, 13 . . . 20, 21 . . . 30, 31 . . . and so forth, translate as 10, 10–1, 10–2, 10–3, . . . 2–10s-1 . . . 3–10s-1 . . . 3–10s, and so forth. English has no comparable pattern for the teens. This difference influences the rate at which children in different countries master the code for generating large numbers although it does not affect rate of learning of the count words for 1–9. American and Chinese children learn these at comparable rates and use them equally well to solve simple arithmetic problems (Miller et al. 1995).

Almost all of the mathematics or arithmetic revealed in the above examples from divergent settings, ages, and cultural conditions map onto a common structure. Different count lists all honor the same counting principles, and different numbers are made by adding, subtracting, composing, and decomposing natural numbers that are thought of in terms of counted sets. The favored mathematical entities are the natural numbers; the favored operations addition and subtraction, even if the task is stated as multiplication or division. The general rule seems to be, find a way to use whole numbers, either by counting, decomposing *N*, subtracting, or doing repeated counting and subtraction with whole numbers. Notions about continuous quantity usually are not integrated with those about discrete quantities, where people prefer to use repeated addition or subtraction if they can. This commonality of the underlying arithmetic structure and reliance on natural numbers is an important line of evidence for the idea that counting principles and simple arithmetic are universal. The reliance on whole number strategies, even when proportional reasoning is used, is consistent with this conclusion.

Understanding the mathematician's zero, negative numbers, rational and irrational numbers, and all other higher mathematics does not contribute to the knowledge base of intuitive mathematics. The formal side of mathematical understanding is outside the realm of intuitive mathematics (Hartnett and Gelman 1998). Even the mathematical concept of a fraction develops with considerable difficulty, a fact that is surely related to the problems people have learning to measure and understand the concept of equal units. Reliance on intuitive mathematics is ubiquitous, sometimes even to the point where it becomes a barrier to learning new mathematical concepts that are related to different structures (Gelman and Williams 1997). A salient case in point is the concept of rational numbers and the related symbol systems for representing them. Rational numbers are not generated by the counting principles. They are the result of dividing one cardinal number by another. Nevertheless, there is a potent tendency for elementary school children to interpret lessons about rational numbers as if these were opportunities to generalize their knowledge of natural numbers. For example, they rank order fractions on the basis of the denominator and therefore say 1/75 is larger than 1/56, and so on. There is a growing body of evidence that the mastery of mathematical concepts outside the range of those encompassed by intuitive mathematics constitutes a difficult conceptual challenge.

See also DOMAIN SPECIFICITY; HUMAN UNIVERSALS; INFANT COGNITION; NATIVISM; NUMERACY AND CULTURE; SCIENTIFIC THINKING AND ITS DEVELOPMENT

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Naive Physics

Naive physics refers to the commonsense beliefs that people hold about the way the world works, particularly with respect to classical mechanics. Being the oldest branch of physics, classical mechanics has priority because mechanical