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Number and language: how are they related?

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Does the ability to develop numerical concepts depend on our ability to use language? We consider the role of the vocabulary of counting words in developing numerical concepts. We challenge the 'bootstrapping' theory which claims that children move from using something like an object-file – an attentional process for responding to small numerosities – to a truly arithmetic one as a result of their learning the counting words. We also question the interpretation of recent findings from Amazonian cultures that have very restricted number vocabularies. Our review of data and theory, along with neuroscientific evidence, imply that numerical concepts have an ontogenetic origin and a neural basis that are independent of language.

Introduction

We have the intuition that our thoughts are inseparable from, indeed dependent upon, the words we use for them. This is especially so for numerical cognition where the claim is that some basic aspects of numerical cognition depend crucially on language, be it knowledge of the vocabulary of counting words or the recursive capacities of syntax and morphology. This argument has been made from neuropsychology, where arithmetical facts are held to be stored in a verbal format; it has been made from neuroimgaing, where numerical tasks appear to activate language areas; it has been made from developmental psychology where counting words are claimed to be necessary for concepts larger than three or four; and, most recently, it has been made from studies of Amazonian tribes whose language lacks counting words. In this article, we challenge all of these claims.

Number terminology

Natural number and arithmetic

Numerical terms and notation can serve a variety of non-mathematical functions. The numeral 4 can denote the position in a sequence, a TV station, a particular football player, a sign of good luck, and so on. For all of these examples there are other ways to refer to the concepts in question. A sequence can be represented by letters and a TV station can be called NBC or ITV. The distinctive numerical concept, and the one that is the focus of this article, is numerosity (the cognitive equivalent of the cardinality) as normally denoted by the natural numbers. Each numerosity N has a unique successor, $N+1$, which is why we can say, for example, that a set of 5 objects includes a set of 4 objects, and so on [1].

Of course, it is possible to estimate or approximate an exact numerosity, and to use continuous variables to do so. For objects of the same type – say, apples – the greater the number of apples, then the greater the amount of apple stuff or weight of apples. It would be possible to estimate the number of a set of apples, or to compare the cumulative magnitudes of two sets of apples on the basis of these continuous variables. However, this does not mean that the mental representation of numerosities must be approximate or continuous. In fact, we know that humans are able to think of both an approximate and exact value for a given set. (See [2] in this series of papers, and [3].)

General vs specific considerations

However, to identify the numerosity of sets larger than about 4, some kind of item-by-item enumeration is required [4]. Typically, this will involve counting using the familiar vocabulary of specialized count words (*one, two,…*) but it could also involve sequenced hatch marks or mapping objects onto a set of known numerosity, such as body parts [5]. For sets with large numerosities, we might not bother, or be able, to enumerate, and instead rely on methods of approximation. Therefore, we need to distinguish possession of the concept of numerosity itself (knowing that any set has a numerosity that can be determined by enumeration) from the possession of re-representations (in language) of particular numerosities.

The relationship between language and number in the brain

At a broad level, the functional relationship between number and language should be evident from their relationship in the brain. Ever since Henschen's extensive case series in the 1920s, it has been known that disorders of language and calculation abilities can occur independently [6]. Recent detailed case studies have confirmed that it is possible for previously numerate adults to have severely impaired language but relatively well-preserved numerical skills [7,8]. Perhaps the clearest reported case is the neurological patient I.H., suffering from semantic

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dementia, whose language comprehension was at chance in most tasks, and whose production was limited largely to stereotyped phrases; however, he scored at or near ceiling on single digit and multi-digit calculation [9,10].

More sophisticated approaches define specific roles for language in adult arithmetic. For example, in the 'Triple Code' model [11], some operations, such as number comparison and subtraction, depend on manipulations over the 'analogue magnitude code', whereas arithmetical facts are stored in the 'verbal code'. Addition and multiplication, which are held to depend more on stored facts than subtraction and division, will therefore be more vulnerable to language disturbances than subtraction and division. However, I.H. was as unimpaired in retrieving stored facts [10].

The Triple Code model also distinguishes exact calculation, which is held to depend on understanding exact numerosities, from approximate magnitudes, which are the responsibility of the 'Analogue Magnitude' code. Evidence in support of this comes from patients who can handle approximate quantities but not exact calculation [12]. It is not clear from presentations of this model how these two codes, analogue and verbal, interact in the process of arithmetic.

Neuroimaging studies make it clear, at least to us, that the crucial brain systems involved in numerical processing are in the parietal lobe, some distance from any classical language areas. In a valuable meta-analysis, Dehaene and colleagues distinguish the bilateral parietal brain areas that have been found to be more active in estimation and approximation tasks (horizontal segment of the intraparietal sulcus and the posterior superior parietal sulcus) from the area more active in exact calculation tasks, and therefore more dependent on language (left angular gyrus). However, the left angular gyrus is not a classical language area, although active in verbal working-memory tasks [13]. It has been known at least from the time of Gerstmann [14], that lesions to this area can cause impairment to exact calculation without concomitant language disorder (see [15] for a review). One neuroimaging study has found that activity in Broca's area is depressed relative to rest during numerical tasks, suggesting that numerical and linguistic processing are even in opposition [16].

Even if the intimate relationship between number and language is not reflected in adult neuroanatomy, a relationship might nevertheless be a requirement for the development of the neural basis of number.

Developmental perspectives

The strong Whorfian claim (of lingusitic relativism) is that language shapes the *development* of numerical concepts. It is rarely clear from the defenders of this position whether they believe that it is the concept of numerosity itself that depends on language, or whether it is concepts of particular numerosities that are entailed. Both Mix, Huttenlocher and Levine [17], and Carey [3] assign to language a causal role in people's acquisition of concepts of natural numbers and their properties. As Carey offers an extensively developed account, we focus on hers here.

Causal dependence on the list of counting words

A crucial assumption for Carey is that infants' numerical abilities involve two different mechanisms, one for small sets of 4 or less and another for larger sets. Parallel individuation of objects, with its built-in limit of 3–4, serves the small number range [3,18–20]. An accumulator process that converts discrete counts into analogue quantities, serves the larger number range. The demonstrations of behavioural discontinuities between small and larger numbers in infants studies justify this dichotomy [18–20], but there are also failures to find the presumed limit in the small number range [21–23]. Carey's account of infants' parallel individuation of objects introduces an unusual use of the notion of 'object files' and their function.

Object files for representing numerosity?

The function of an object file is to 'point' to a visual object and integrate the perception of the properties of that object, such as size, shape, color, and so on [24,25]. If a child believes that the word *two* referred to a particular set of two object files, it would presumably be useable only in connection with the two objects they pointed to. It would be a name for that pair of objects, not for all sets that share with that set the property of twoness. A particular set of pointers cannot substitute for (is not equal to) another such set without loss of function, because its function is to point to a particular pair of objects, whereas the function of another set of pointers is to point to a different pair. There is no reason to believe that there is any such thing as a general set of 'two pointers', a set that does not point to any particular set of two objects, but represents all sets of two that do so point. Any set of two object files is an instance of a set with the 'twoness' property (a token of twoness), but it can no more represent twoness than a name that picks out one particular dog (e.g. *Rover*) can represent the concept of a dog. Being able to abstract away from the particularities of the transient representation of items of current attention, to represent their numerosity, seems to be a *precondition* for making use of this system to bootstrap concepts of number rather than a consequence of using it.

It is most unusual to use a processing constraint – the number of objects whose features can be bound in a transient store – as the basis for the development of a system of knowledge. Nevertheless, let us grant the case for sake of argument that there is a discontinuity in infants' numerical discriminations and that the small number range uses Carey's version of an object-file mechanism.

Language first?

Carey's argument goes forward as follows: once the first three or four count words are memorized they can be treated as separate from the non-precise quantifiers of *some, few, many.* The meaning of the count words is induced from the fact that they map to different sized sets of object files and they are used in a fixed order, with each successive word coming to represent a larger object-file set. This allows for the integration of the infants' numerical representations of small numbers and their short list of memorized count words. Language serves as the mechanism for creating a restructuring of the non-verbal notions of number and, according to Carey, does so as follows.

A child comes to recognize the ordering of the referents of *one, two, three* and *four* because a set of two active object files has as a proper subset a set of one object file, and so on. The child infers that addition applies to the things referred to by these words because the union of two sets of object files yields another set of object files (provided the union does not create a set greater than 4). This is the foundation of the child's belief in the successor principle: every natural number has a unique successor. The crucial bootstrapping comes when the child realizes that it is possible to go beyond the narrow limits of parallel individuation of objects, to sequences of objects that are not all within the span of immediate apprehension. This move is prompted by the use of count words for sets of more than 4 objects. That is, the child having established the correspondences for $N \leq 4$ – the word '*one*' refers to the state of the object-file system where there is just one object; the word '*two*' refers to the set of two objects (which is one more than one); and so on – makes the inference that other syntactically equivalent words (*five*, *six*, and so on) refer to numerosities greater than the immediate span of apprehension.

This account entails that meanings are first assigned only to the words that have been mapped onto states of the object-file system. So, when the child is unable to give reliably, on request, four, five, or six objects, it has to be assumed, on this model, that he or she knows nothing about the mapping, and that these words will refer indiscriminately to numerosities greater than 3. However, there is evidence that although children at this stage of development are unable to give exactly what number is designated by '*five*', they know that only number-changing manipulations of the target set will require a change of number word [22,23]. Although young three-year-old children fail a 'Give *N*' task when *N* is greater than 3, they can succeed on prediction and checking tasks with set sizes ranging from 1–5 [23]. The implication is that young children understand how numbers work before they have fully mastered the mapping from particular numbers to particular numerosities.

Carey's position makes clear predictions about the numerical capacities of children growing up in cultures where there are few or no counting words, and especially where there is no linguistic means for creating ever larger number names. One prediction is that the children will not develop 'true' understanding of the natural numbers because it is the system of counting *words* that is crucial. Other semiotic means for representing number, such as using body-parts [5,26], tallying, or drawings in the sand (D.P. Wilkins, Ph.D. thesis, Australian National University, 1989) are not mentioned as enabling the development of numerical concepts beyond 3 or 4, the limit of the object-file system of parallel individuation. We concur with Harris [27,28] that it is a mistake to dismiss these alternative re-representation systems.

Amazonian mysteries

The Tououpinambos

One of the earliest accounts of the numerical abilities in people with restricted number vocabularies comes from the English philosopher John Locke [29], who wrote:

'Some Americans I have spoken with (who otherwise of quick and rational parts enough) could not, as we do, by any means count to 1000; nor had any distinct *idea* of that number.' He was not referring to the founders of the USA, but to the Tououpinambos, a tribe from the depths of the Brazilian jungle, whose language lacked number names above 5. A system of number names was not, in Locke's view, necessary to have ideas of larger numbers, because, he says, we construct the idea of each number from the idea of *one* – 'the most universal idea we have'. By repeating 'this idea in our minds and the repetitions together… by adding [the idea of] of one to [the idea of] one we have the complex idea of a couple'. And so on. Thus, he says, concepts of numbers are independent of their names. Indeed the 'Americans' can reckon beyond five 'by showing their fingers, and the fingers of others who were present.' Even the idea of infinity, he proposes, is simply a consequence of understanding that we can repeat the procedure for adding one as many time as we wish. Possessing a system of number names is useful for keep track and communicating, but not necessary for having the ideas of distinct numerosities and their infinity.

Locke's point was that number names 'conduce to well-reckoning' by enabling us to keep in mind distinct numerosities. That is, the possession of a system of number names can be helpful in learning to count and to calculate, but is not necessary for the possession of numerical concepts. Recent reports about the Pirahã [30,31] and Mundurukú Amazonian Indians [31] provide informative test cases of Locke's conclusion.

The Pirahã and Mundurukú

The Mundurukú language uses the count words for 1, 2 and 3 consistently, and 4 and 5 somewhat inconsistently [31]. The Pirahã do not even use the words for 1 and 2 consistently [30]. How would members of these groups perform on various non-verbal tasks involving numerosity? The amazing result was that both groups succeeded on non-verbal number tasks that used displays representing values (in one study) as large as 80.

The findings on the Mundurukú are especially noteworthy, because an elegant research design was used that incorporated the fact that some Mundurukú adults and children are bilingual (in Portuguese). In the study [31], there were groups of adults and children who were monolingual and groups who were bilingual. The children's groups were divided into younger (<5 yrs) and older, as well as those who had had language instruction and those who had not. Finally, there was a control group of French adults. The various groups were asked to point to the more numerous of pairs of dots, whose numerosity could be as large as 80. *All* the groups showed the effects of number size and number difference in number comparison tasks [32]. There was no effect of language or schooling amongst the Mundurukú. The data for the number-difference effect for the Mundurukú groups and the

French adult control groups were extremely close. Although the Pirahã study did not have as many subjects and conditions, the results were comparable. They too were able to engage in a variety of comparison and non-verbal arithmetic tasks – despite their lack of any clear number word vocabulary. The Pirahã solved the problems in ways that overlap extensively with those used by English- and French-speaking individuals [32].

The key claim of defenders of the 'language thesis' is that language is necessary for mental representation and manipulation of numerosities greater than 4. In the Mundurukú study, exact addition and subtraction problems using sets of objects were tested. According to the theory, the participants who were bilingual, and therefore knew the counting words of Portuguese, should have performed like the numerate French controls, or at least more like the French controls. But this was not the case. Both adults and children performed exactly like the monolingual speakers [31]. The Mundurukú continued to deploy 'approximate representations… in a task that the French controls easily resolved by exact calculation' [31]. Why, we may ask, did the bilingual participants not use their Portuguese counting words? Mundurukú culture differs from Western culture in innumerable ways, and it certainly uses numbers far less often than we do. It remains possible that one or more of these many differences were responsible for the differences in performance, and not just the lack of a counting vocabulary.

This evidence from cultures with very limited number vocabulary does not convince us that differences in performance can be explained in terms of language rather than other aspects of culture (see also Box 1). Of course, it remains possible that Pirahã and Mundurukú have few number words because numbers are not culturally important and receive little attention in everyday life.

Causal dependence on the recursive capacities of language

Bloom proposed that children's initial counting is embedded in natural language as a result of their learning relevant distributional facts [33]. As they learn more and more count words, they infer that there are more count words. With enough experience, they infer that natural numbers are discretely infinite. Hauser, Chomsky and Fitch [34] offer a statement as to how this might work. This is that the recursion involved in the mathematical idea of discrete infinity derives from a recursive capacity that is the foundation of, and unique to, human languages. Carey [3] makes the same in-principle assumption. We are puzzled about this claim. Because the mental magnitudes that represent larger numbers are additive, the recursive infinity of magnitude is already entailed.

Instead, we are making the case that understanding recursive infinity is not derived from language at all. To illustrate this, in one study [35] children aged 5 years to 8 years 6 months (written 8;6) were asked to participate in a thought experiment about the effect of repeated additions or 'counting-on' from what they said was a very large number. Many of the younger children showed that they were still finding the language to express themselves, as was the case for D.A. (5;9). When asked whether adding would yield a higher number, she replied yes, because '*You still put one and they get real higher*'. Some of the older children were explicit about the possibility of inventing count words to satisfy the successor principle for natural numbers. For example, A.R. (7;3) said there is no end to the numbers, '*Because you see people making up numbers. You can keep making them, and it would get higher and higher*'. [35].

There are several language experts who hold that, despite the fact that some speakers of languages have restricted number terms, they can easily acquire them. Dixon quotes Kenneth Hale, an expert on Warlpiri (a language with terms for *one, two, few, many*): 'the English counting system is almost always instantaneously mastered by Warlpiris who enter into situations where the use of money is important (quite independently of formal Western-style education.' ([36], p 108). A potent example of the rapid uptake of the idea of discrete infinity comes from Saxe [5], who studied the Oksapmin of New Guinea, a group who used use a fixed number of sequential positions on their body as 'count words'. There came a time when some of the men were flown out to work on plantations and received money for their labor. Within 6 months, the Oksapmin had introduced a generative counting rule [5]. It is hard to see how such rapid learning of a new vocabulary for abstract objects like numerosities could proceed so quickly, if the learners did not already possess the concepts.

Finally, although the Pirahã do not use numerals in their everyday life, Everett (personal communication) reports that it is easy to teach their children to count in 'Portuguese' if the pronunciation rules are adjusted to fit the phonetics of Pirahã and the teaching is embedded in the everyday task of stringing beads.

Box 1. Questions for future research

- How do varieties of language, especially varieties of number-naming systems, promote or inhibit acquisition of basic numerical concepts?
- Is there a critical or sensitive period for acquiring numerical concepts?
- How do developmental language disabilities affect the acquisition of arithmetical skills? Can there be a selective deficit of arithmetical development?
- Why do mathematically literate individuals continue to use the non-verbal, approximate numerosity system?
- How does the neural network for numerical processing develop from infancy to adulthood?

Conclusions

Cognitive development reflects neural organization in separating language from number. Indeed, the ontogenetic independence of the number domain has been argued vigorously by the authors of many previous publication looking at both normal [4,35,37] and abnormal [38–40] development of numerical abilities. It would be surprising if there were no effects of language on numerical cognition, but it is one thing to hold that language facilitates the use of numerical concepts and another that it provides their causal underpinning. That very young children's knowledge of count words is incomplete is far from surprising. They constitute a serial list of sounds: there is nothing about the sound *one* that predicts that the sound for 'two' will follow, and so on. In addition the young child has to master the extensive coordination requirements of counting. Locke put the matter elegantly more than three hundred years ago: 'Children, either for want of names to mark the several progressions of numbers, or, not having yet the faculty to collect scattered *ideas* into complex ones and range them in a regular order and so retain them in their memories, as is necessary to reckoning, do not begin to number very early, nor proceed in it very far.'

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