Cognition, 13 (1983) 343–359

Preschoolers' counting: Principles before skill*

ROCHEL GELMAN
ELIZABETH MECK
University of Pennsylvania

Abstract

Three- to 5-year-old children participated in one of 4 counting experiments. On the assumption that performance demands can mask the young child's implicit knowledge of the counting principles, 3 separate experiments assessed a child's ability to detect errors in a puppet's application of the one-one, stable-order *and* cardinal *count principles. In a fourth experiment children counted in different conditions designed to vary performance demands. Since children in the errror-detection experiments did not have to do the counting, we predicted excellent performance even on set sizes beyond the range a young child counts accurately. That they did well on these experiments supports the view that errors in counting—at least for set sizes up to 20—reflect performance demands and not the absence of implicit knowledge of the counting principles. In the final experiment, where children did the counting themselves, set size did affect their success. So did some variations in conditions, the most difficult of which was the one where children had to count 3-dimensional objects which were under a plexiglass cover. We expected that this condition would interfere with the child's tendency to point and touch objects in order to keep separate items which have been counted from those which have not been counted.*

Although preschoolers count only small sets accurately, Gelman and Gallistel (1978) propose that preschoolers' counting is governed by the *implicit* knowledge of five counting principles. These are: (1) the one-one principle—every item in a display should be tagged with one and only one unique tag; (2) the stable order principle—the tags must be ordered in the same sequence across trials; (3) the cardinal principle—the last tag used in a count

*Support for this work came from NSF grants BNS-8004885 and BNS-8140573. We thank Ellen Markman for suggesting a version of experiment 3 and C. R. Gallistel and Jim Green8 for comments on an earlier version of this manuscript. Request for reprints should be sent to Rochel Gelman, Psychology Department, University of Pennsylvania, 3815 Walnut Street, Philadelphia, PA 19104, U.S.A.

0010-0277/83/$5.60

sequence is the symbol for the number of items in the set; (4) the abstraction principle—any kinds of objects can be collected together for purposes of a count; and (5) the order-irrelevance principle—the objects in a set may be tagged in any sequence as long as the other counting principles are not violated. The first three of these principles define the counting procedure; the fourth determines the types of sets to which the procedure may be applied; and the fifth distinguishes counting from labelling. Gelman and Gallistel suggest that children know these principles at a very early age but have difficulty putting them into practice with larger sets.

We refer to knowledge young children have about counting as *implicit*, as opposed to explicit, for much the same reasons such a distinction is made in the psycholinguistics literature. Very young children are granted implicit knowledge of language rules well before they are said to have explicit knowledge of their grammar (e.g., Clark and Clark, 1977; Gleitman *et al.*, 1972). Explicit knowledge involves the ability to say why a given sentence is agrammatical or not. This metalinguistic ability develops around five or six years of age. Long before these young children have implicit knowledge of the language they speak because: they utter novel sentences that honor rules of the language; they self-rehearse and self-correct their sentences; they produce overgeneralizations that accord with a rule-governed knowledge of the grammar but do not accord with what the child has actually heard; and they can recognize violations of rules in the speech of others (Clark and Clark, 1977).

When we distinguish between implicit and explicit knowledge of the counting principles we mean to distinguish between the ability to verbalize or state the counting principles and the ability to demonstrate that one's behavior is systematically governed by the principles (see also Greenȯ *et al.*, 1982). Gelman and Gallistel offered two kinds of evidence for the young child's implicit knowledge of counting principles. First they noted the orderly count sequences observed in young children. Preschoolers do not invariably count correctly, however. There are errors and the nature of the errors served as the second source of evidence for the Gelman-Gallistel conclusion. Errors which were observed were classified as performance errors or idiosyncratic 'errors'. To show why the latter errors are especially telling, it is best to give an example. Some 2-year-old children count with what Fuson and Richards (1979) calls nonstandard and we call idiosyncratic count lists. Consider the child who says '1-2-8' when pointing to a 3-item array and '1-2-8-10' when pointing to items in a 4-item array and '10' when asked *how many* there are in the latter case. The child meets the requirements of the counting principles with an unconventional list of her own. Since Gelman (1977) first reported such a finding, parents have been coming to us with the list their child once used. Two favorite ones reported by parents are '1-2-3-4-5-6-7-h-i-j-k' (the switch in lists is most likely due to an acoustic confusion error between 8- and h) and 'red, yellow, blue' (even where the objects were all the same color).

The invention and correct use of idiosyncratic lists is hard to explain unless appeal is made to some implicit rules that guide the children's search of their environment for a list with which to count. This argument is the same one used to account for overgeneralization errors, e.g., runned, unthirsty, in a child's speech. The occurrence of such novel but lawful count sequences makes it necessary to postulate an implicit set of rules.

Gelman and Gallistel also noted that young children have a tendency to generate count sequences without prompting and self-correct the errors they make in the course of these self-generated sequences. Again, it is hard to imagine a child doing this unless the child refers its counting performance to some set of internally represented principles. Yet, it has been argued (Siegler, 1979; Sternberg, 1980) that Gelman and Gallistel grant young children too great a competence when they impute principles to very young children. The experiments reported here buttress the data base for the idea that counting principles guide the counting behavior of young children.

One reason to challenge our view of preschooler's competence is that their ability to count accurately is restricted to small set sizes. Poor skill at counting set sizes larger than 5 is taken as evidence for the conclusion that many of the component counting principles have yet to be learned (e.g., Mierkiewicz and Siegler, 1981). Gelman and Gallistel maintain that children count small set sizes better than they count large set sizes because the latter present too much of a performance demand on the children. If so variations in performance demands should produce variations in success on counting tasks. By contrasting performance in easy tasks (error detection) and hard tasks (counting items under a plexiglass cover) we hoped to show that performance problems and not faulty principles explain many of the errors in the counting behavior of very young children.

The error detection studies

In the error detection experiments children watched a puppet count and told the experimenter whether the puppet was right or wrong. Thus children did not have to generate the counting performance, they only monitored it for conformance to the principles. When the child does the counting herself, she must both generate and monitor the performance. Hence, an error detection task should be easier than a standard counting task.

There were three error detection experiments. One focussed on the one-one principle, one on the stable-order principle, and a third on the cardinal principle. In all cases, children were asked to help teach a puppet to count by indicating whether a particular count sequence was correct. In all cases the experiment proper began only after the experimenter had spent time playing with the children in their classrooms and then taken each child individually to play with commercially bought toys in the experimental room.

The one-one study

Children who participated in this study were given three kinds of trials; correct, in-error, pseudoerror. On the two correct trials the puppet correctly counted a linear array from beginning to end. The two in-error trials were of two types, an item was *skipped* or an item was *double counted.* The two pseudoerror trials were included for purposes of comparison with a study which was being run by Mierkiewicz and Siegler (1981). On one of these trials, a puppet started counting in the middle of a linear array, then continued to the end of that row before returning to the beginning of the array for the remaining to-be-counted items. On the other trial, the puppet confronted a row of alternating red and blue chips and counted the red ones before doubling back to count the blue ones.

Subjects

The children in this study were 12 3-year-olds (median age, 3 yr. 7 mo.; range, 3 yr. 1 mo.–3 yr. 11 mo.) and 12 4-year-olds (median age, 4 yr. 5 mo.; range, 4 yr. 1 mo.–4 yr. 10 mo.). They were from one of two day-care centers or one of two nursery schools in the Greater Philadelphia area. The population served by these schools tends to be middle-class; although the children in attendance at these schools are quite heterogeneous with respect to race or ethnic background.

Procedure

To start the experiment, children were shown a row of red and blue objects and then told: "This is my friend, Mr. Horse (Lion) and he would like you to help in playing the game. Mr. Horse is going to count the things on the table but Mr. Horse is just learning how to count and sometimes he makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes counting if it was OK to count the way he did or not OK. So remember you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong".

Each child was given 6 trials per set size: two correct trials, two in-error trials and two pseudoerror trials. The 4-year-olds were tested on set sizes of 6, 8, 12, and 20. The 3-year-olds were tested on only set sizes 6 and 12 because we assumed that even this many trials might tax their attention span. (To catch the puppet skipping or double-counting but one item, the children had to pay close attention). The order of the six trials within a set-size block was random as was the order in which a set size was presented. The session lasted 15 minutes.

In pilot work we found that some children would declare a trial incorrect or correct before the puppet had finished that trial. When this happened in the present study the child was told to wait until the puppet finished and then the experimenter restarted that trial.

Table 1. *Number of children who meet the 75% criterion as a function of problem type for set sizes 6 and 12**

Age group	Problem type in the one-one experiment		
	Correct	Pseudoerrors	Incorrect
(N = 12)			
3-years	12	11	9
4-years	12	10	10

*The 4-year-olds' trials for set sizes 8 and 20 are not included for this summary.

Results and discussion

Nearly all of the three- and four-year-old children detected skipping and double-counting errors in the puppet's counting on a majority of the trials in which such errors occurred. Table 1 shows that 9 of 12 three-year-olds caught the puppet's mistake on at least 3 of 4 trials, as did 10 of 12 four-year-olds. There was no effect of set size in either group.

Table 2 gives the mean percent correct answers per child as a function of age (3 years *versus* 4 years) and type of trial (correct, pseudoerror, in-error). An ANOVA on trials with set sizes of 6 and 12 (the sizes used with both age groups) showed that the rightness of the child's assessment of the puppet's

Table 2. *Average percent correct responses on each trial type in the one-one error detection task*

Age/Set sizes	Type of trial		
	Correct	Pseudo	Error
3-yrs.—6 & 12	100	96	67
4-yrs.—6 & 12	100	96	83
4-yrs.—all	100	95	82

counting depended on the type of trial, ($F2,44 = 21.4$, $p < 0.001$). This reflects the fact that the children made no errors on correct trials. When they erred it was when judging error trials. There was also an interaction between age and type of trial, the younger children being worse than the older at spotting errors, but not at judging correctly the other two kinds of trials ($F2,44 = 3.22$, $p < 0.05$). The ANOVA showed no main effect of age or set size. A separate ANOVA for the 4-year-olds revealed no effect of set size ($F3,33 = 1.24$, $p < 0.31$) where set sizes were 6, 8, 12 and 20. And as before, type of trial did matter ($F2,22 = 5.70$, $p < 0.01$). Otherwise, there were no reliable effects.

Further support for the conclusion that the children knew that skipping and double-counting are errors comes from their explanations. All but two 3-year-olds and one of the 4-year-olds offered at least one argument that the puppet 'missed one' or 'did it again' (double-counted). Although 10 of 12 three-year-olds and 11 of 12 four-year-olds offered an explanation on at least one of the in-error trials, they did not do so very often; explanations were *not* obtained on a majority of error trials. Comments on the pseudo-error trials, while also infrequent were usually illuminating e.g., "It's a little bit funny to count that way" or "Yes, that's OK—you count this one, the other ones (points to reds) and on the way back you count this blue one and this blue one..." The explanations quoted here reflect a common feature: When they did explain a trial, children could say what kind of error was made or what was peculiar about the counting (in the case of pseudo-error trials); however, none of the children articulated the one-one principle itself, i.e., that each and every object needed to be tagged once and only once with one unique tag. This is consistent with the hypothesis that explicit knowledge is relatively late to develop. (cf., the literature on meta-memory, e.g., Flavell and Wellman 1977.).

We find that 3- and 4-year-old children are able to distinguish between erroneous and correct applications of the one-one count principle. Four-year-olds are better at this than are 3-year-olds. Both groups treat a pseudoerror trial as just that, peculiar but not erroneous. Most importantly, set size has little effect, if any, on this ability. In contrast, it has a very strong effect on children's ability to do the correct count themselves (Gelman and Gallistel, 1978; Fuson and Richards, 1979; and the final study in this paper).

We were surprised at how well the children handled the pseudoerror trials. This is because Mierkiewicz and Siegler (1981), in a fairly comparable study reported that even 5-year-olds failed to recognize what we call pseudoerrors as such. There are two possible reasons for the discrepancy. First, children in the latter experiment were tested on 72 trials over-all and hence possibly paid less attention; ours received many fewer trials. Also, recall that when the child said the puppet had been correct or incorrect before waiting until the puppet finished, the trial was started again. If we had not followed this rule pseudoerror trials could have been treated as 'mistakes' simply because the child did not wait out the trial. It appears that Mierkiewicz and Siegler (1981) did not start a trial over whenever a child judged it correct or incorrect before hearing it to the end.

The fact that variations in set-size had little effect on success rates deserves comment. Children as young as these seldom count correctly set sizes even as small as 8 (e.g., Gelman and Gallistel, 1978). In the present experiment, the children did not do the counting themselves; they commented on what the puppet did. The experiment did more than investigate the ability to detect errors in the application of the one-one principle. It provided evidence that children who do not have to do the work themselves are able to recognize when a count trial is correct or not—even for sets well beyond what they themselves can count. This implies that these young children know in principle what it is to count rather large sets but they have difficulty putting their principles into practice. Of course, it is possible that children would have difficulty with larger set sizes, especially the 3-year-olds. Hence the decision to try to test all children on set size 20 in the following 2 experiments.

The stable order study

Children in this experiment were given five trials per set size, two correct ones and three during which the stable order principle was violated in one of three ways: The violations were (1) use of a list wherein the conventional order of two items was reversed, e.g., 1, 2, 4, 3, 5, 6: (2) a use of a randomly-ordered list for the set size in question e.g., 2, 1, 5, 3, 4; and (3) use of a list which skipped one or more tags in a standard count list e.g. 1, 2, 3, 5, 6, and 1, 2, 3, 4, 7, 8, 9.

Subjects

The children in this study were 12 3-, 12 4- and 12 5-year-olds. The respective medians and ranges of age were 3 yr. 7 mo. (3 yr. 1 mo.–3 yr. 11 mo.); 4 yr. 5 mo. (4 yr. 2 mo.–4 yr. 7 mo.), and 5 yr. 5 mo. (5 yr. 0 mo.–5 yr. 11 mo.). Children were drawn from the same sample as that used for the first experiment.

Procedure

The instructions were the same as those used in the first experiment. The objects were small trinkets which varied in type and color. The set sizes were 5, 7, 12, and 20, all of which were displayed in a row. The five trials (2 correct, 3 incorrect) for each set size were run in random order, and the order in which a child encountered the block of 5 trials for a set size was also randomized. In trials where number names were omitted, whether 1, 2, or 3 were dropped was randomly determined: so was the locus of omission within the sequence.

Results and discussion

The overall tendency for children to indicate when the puppet made a stable-order error in counting is shown in Table 3. Nearly all children at all ages were able to say that a correct use of the conventional list was just that. Similarly, the children did well at calling an error an error.

Table 3. *Mean percent correct judgments of correct or incorrect on stable-order trials as a function of age and trial type*

Age (N = 12)	Correct order trials	Incorrect order trials
3	96	76
4	100	96
5	98	97
Mean	98	89

An analysis of variance revealed a significant effect of age ($F_{2,33} = 6.34$, $p < 0.01$), error-trial type ($F_{2,66} = 7.73$, $p < 0.001$) and age X trial type interaction ($F_{4,66} = 4.56$, $p < 0.01$). There was no reliable effect of set-size ($F_{3,99} = 2.24$, $p = 0.09$) or other interactions.

The significant effect on error trials for the type-of-trial variable means that the three-year-olds had trouble detecting jump-in-the-list errors. In this younger group, only 60% of the trials in which number names were omitted were correctly identified as error trials. In contrast, the 3-year-olds caught 80% of the trials which involved a reversal of a pair of items in the conventional count list, and 90% of the random strings of number words. We find it most interesting that the jump-in-list trials were so hard for the 3-year-olds for this is just the kind of error that occurs in young children's own lists when they are summoning them up in the course of a count (Fuson and Mierkiewicz, 1981). Of the 3 error types this can be taken as the less deviant form because the resulting lists are still ordered according to the conventional list. Hence it is possible that some of the 3-year-olds in the study had yet to accommodate fully their own counting lists to the conventional list. Still, they were able to determine whether a puppet's count was correct or not as a function of the other 2 error types.

Once again we find no effect of set size. This nonresult is consistent with the position that the young child's counting errors are due more to excess demands on processing activities than to a faulty understanding of counting principles.

The cardinal study

The instructions in this study were somewhat different owing to the nature of the principle being tested. Children were asked to indicate whether the puppet gave the right answer when, after counting, it was asked "how many?". They were also encouraged to tell the puppet what the right answer was. Otherwise the procedure was as in the stable-order experiment. Hence there was a block of five trials per set size. Set sizes were 5, 7, 12 and 20. On two of the trials the puppet gave the correct cardinal number; on three it gave an erroneous number. The order of set-size blocks and within-block trials was randomized as before.

The error trials represented 3 deviations from the principle that the last tag in a count sequence represents the cardinal value n of a display. They were: (a) to give the nth + 1 value in response to the *how many* question; (b) to answer $a < n$, a number preceding n in the list of n tags and (c) to offer the color or some other irrelevant designation of the nth object, e.g., a boat. We have observed that some severely retarded children produce the last error. They and very young children also produce the $a < n$ error.

Subjects

Twelve 3- and twelve 4-year-olds served as subjects. Their median age and respective range of ages were: 3 yr. 7 mo. (3 yr. 2 mo.–3 yr. 10 mo.) and 4 yr. 5 mo. (4 yr. 4 mo.–4 yr. 11 mo.). They attended the same schools as those in the above experiments.

Results and discussion

The error detection task was especially easy for the children. Three-year-olds gave the right answer on an average of 85% and 96% of their error and correct trials, respectively. For 4-year-olds, the comparable figures were 99 and 100%. An analysis of variance revealed a significant effect of age. Otherwise there were no reliable effects in an analysis of age, set size and error-type.

Children in this experiment were asked if they wanted to change the puppet's answer and if so to go ahead. The 4-year-olds attempted to correct 90% of the puppet's error trials; 93% of the answers on these attempts were correct. The 3-year-olds attempted a correction on 70% of the puppet's error-trials and 94% of these corrections were right. The children in this experiment knew not only that the puppet had erred; they knew what it had done wrong and could correct the mistake. (By contrast, the nonmusical may often know that a singer has erred but not know how and not be able to sing it the way it ought to have been sung.)

The findings in these error-detection experiments support the hypothesis that children as young as three have implicit knowledge of the counting principles. That we failed to show an effect of set size in all of the studies is consistent with the Gelman and Gallistel hypothesis that knowledge of the counting principles guides the child's interaction with her environment and that it is the perfection of superior procedures that underlies the development of counting within the preschool period rather than the emergence of new or firmer principles. Wilkinson's (1981) treatment of the nature of partial knowledge provides another way of stating this hypothesis.

Wilkinson makes a distinction between what he calls the *variable* and *restrictive* application of a given domain of knowledge. Knowledge that is variable is that kind of knowledge which varies as do the performance demands of the task. In contrast, restricted knowledge may be revealed in every task but only for a limited set of stimuli. In the case of number, Wilkinson points out that the distinction will be between whether the children's knowledge is only as extensive as the set sizes on which they never err or whether the knowledge is fragile, albeit for a wide range of set sizes. In the latter case, success on a task will vary as a function of set size only when there is reason to believe that the performance demands of a task are too great. The next experiment was based on such considerations.

The plexiglass study

Young children are prone to point to, touch and/or move items as they are counted (e.g., Gelman and Gallistel, 1978; Fuson and Richards, 1979). This presumably serves their efforts to keep separate the items which they have already counted and those remaining to-be-counted. Hence, 3-dimensional items which can be touched, pointed to, and/or moved should be easier to count than 3-dimensional items which cannot be touched, etc. To find out if this was the case, we asked the same children to count the exact same set of 3-dimensional objects twice; once without a cover and once with a cover made out of plexiglass. The same children also participated in two further conditions. In both of these, the items were a heterogeneous collection of stickers on a card, i.e., the items were 2-dimensional. Since children would be able to point to and touch the items in these 2-dimensional (2D) displays, we thought that they would do better in these conditions than they would in the plexiglass one. The two 2D conditions differed with respect to the degree of spacing there was between each of the items on the cards. The items in the 2D-*near* condition touched each other. Those in the *far* condition did not.

Subjects

Eighteen 3- and 18 4-year-old children were the subjects in this experiment. Their respective median ages were 3 yr.–4 mo. (range 3 yr. 0 mo.–3 yr. 10 mo.) and 4 yr. 5 mo. (range 4 yr. 0 mo.–4 yr. 9 mo.).

Procedure

As above, we used a repeated subjects design for this experiment. Children were tested on each of the 4 main conditions with all set sizes (3, 4, 5, 7, 9, 11, 15, 19) over two sessions. (In the case of the 3-year-olds, many took 3 sessions.) The nature of the stimulus type (2D or 3D) was counterbalanced across the two sessions. The order in which the two conditions for each kind of stimulus were encountered was also counterbalanced. There were 3 trials for each set size within a condition and thus 12 trials per set size all together. Within a condition, the order of set sizes was random, with the provision that all 3 trials for a given set size within the condition were run one after the other.

We anticipated that the children in this study might find it tedious; hence, the experimenter interrupted a session with little breaks for playing with other toys or just chatting. In addition, the testing involved a 3-way conversation between the child, the experimenter and a puppet. We have found the latter manipulation very useful in cases where the child might not want to continue (e.g., Bullock and Gelman, 1977).

Stimuli for the 3D conditions were a variety of magnetized objects which are used to hold up memos on metal surfaces like a refrigerator. They varied in color, size and shape, and were approximately 4 cm high. Stimuli for the 2D conditions were colored stickers which were of about the same size and which represented a variety of colors, types, and so on. The 3D stimuli were arrayed on a large metal plate (14.5 in. in diameter) which in turn was placed on a base. Since the plate was metal, the magnetized objects stayed in place. Since the base rotated, the display could be turned around between the count trials for a given set size. And since the base stood off the ground, this could be done from beneath the display. The plexiglass cover, when in place, sat over the plate on the base.

The stimuli in the 3D conditions were arranged in a row or a crescent (for the larger set sizes). The row of stickers in the 2D conditions either touched each other (*near*) or were separated by 1.5 cm (*far*). For the 3 trials within a set size, the displays were rotated 90° or 180°.

The child's task was to count the number of objects and then, while the array was covered with a black cloth, indicate the cardinal value of that display. All sessions were video-recorded.

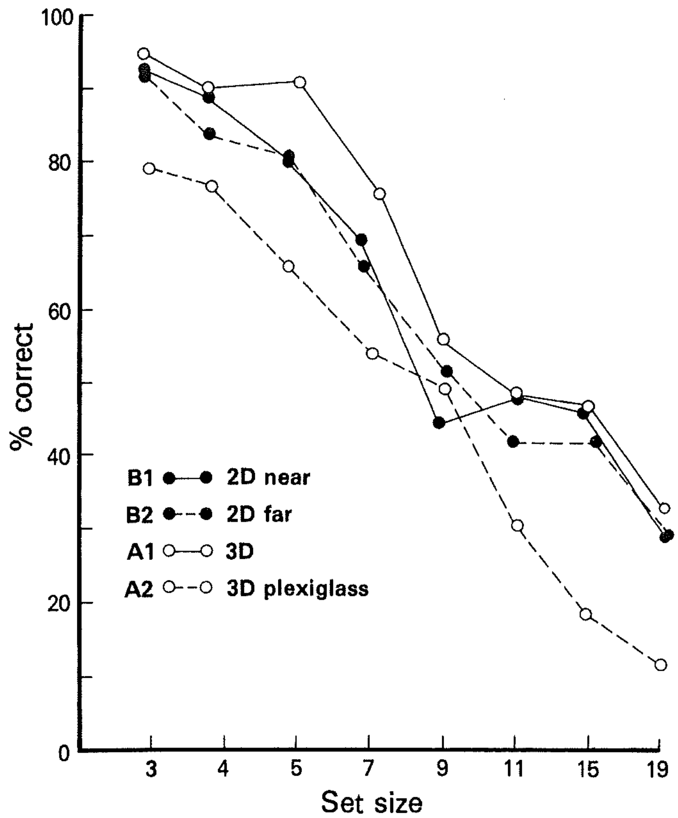
Each of the children's trials were scored as correct or incorrect. Following Gelman and Gallistel, a child could be scored correct if he or she used an idiosyncratic list, as long as that list was used consistently. Otherwise, to receive credit for the trial, the child had to use all 3 how-to-count principles correctly. If a trial and/or set size was not run the child received a zero for that trial (set size).

Results and discussion

The results are shown in Figures 1 and 2.

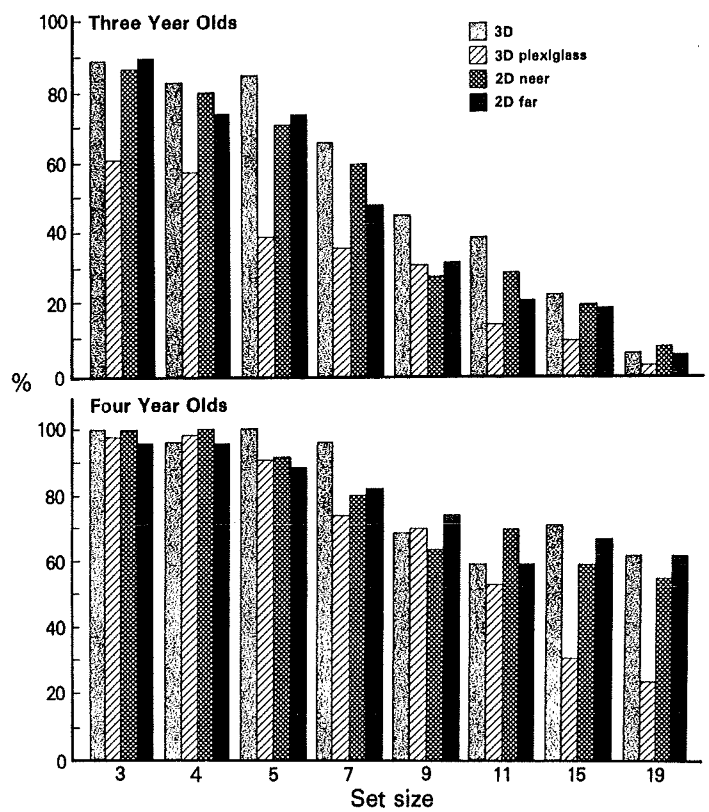
Not surprisingly, an ANOVA showed that 4-year-olds did better than the younger children and larger set sizes were harder than smaller set sizes (in both cases $p < 0.001$). That there was a significant interaction between set

Figure 1. *Percent correct counting trials as a function of set size and condition in plexiglass experiment.*



size and age ($p < 0.001$) reflects the fact as shown in Figure 1, that the effect of set size appears at lower values for the younger children. There was also an effect of the conditions under which the child had to count. This effect ($p < 0.001$) was due mainly to the difficulty children had with the plexiglass condition which was *harder* than the standard 3D condition. The difference between the two 2D conditions was not significant. It is noteworthy that the plexiglass condition was more difficult than either of the 2D conditions. Theories which maintain that children are late to apply their number knowledge to two dimensional items because they are more abstract stimuli than are three dimensional ones (e.g., Gast, 1957; Klahr and Wallace, 1976) are challenged by such a finding.

Figure 2. *Percent correct counting trials as a function of age, set size and condition in plexiglass experiment.*



The ANOVA also showed an expected two-way interaction between age and set size (p = 0.001) and an expected three-way interaction between age, set size, and condition ($p < 0.001$). The younger children with more presumably fragile and less practiced performance skills begin making mistakes at smaller set sizes that the older, more skillful, children can handle: hence, the two-way interaction. The older, more skillful, children also can handle the smaller sets under all conditions—including even the most difficult, plexiglass one. They first begin to falter when they encounter the larger sets under the hardest partitioning conditions: hence the three-way interaction between age, set size, and counting condition.

Overview

The above findings offer strong support that children as young as 3 years know the principles that a procedure must conform to in order to be a valid counting procedure, although of course, they cannot articulate these principles. The range of set sizes to which the children know that these principles apply is much greater than the range that they can successfully count. Indeed, there is no evidence that the children recognize any upper bound on the range of set sizes to which these principles apply. The development of counting in the early years would appear to be mostly a matter of developing procedures that implement these principles and acquiring skill in the carrying out of these procedures.

We do not mean to suggest that the development of counting in children beyond the age of 2 or 3 years does not involve the emergence of any new principles. We have argued elsewhere that at least three new principles of far reaching significance emerge. The emergence of all of these principles may be induced in the course of carrying out the kinds of procedures dictated by the counting principles. We offer three examples of such development.

The counting of large sets requires a lengthy and stably ordered list of number words, and humans have difficulty forming such lists. The first consequence of this difficulty is that very few five-year-olds can count to 20 reliably and need to work hard to achieve this ability. If learning to count from 3 on up to 20 were any predictor of the difficulty of learning to count from 20 on up to a thousand, it is safe to say that very few humans would ever learn to count to a thousand. But humans never construct a list of a thousand number words by brute force—as could a computer. They invariably fasten on a generative scheme involving one or more number-generating bases (Zaslavsky, 1973; Saxe and Moylan, 1982). Such schemes permit the generation of indefinitely long lists of numerlogs[1] by the lawful combination of the relatively few numerlogs in a base set. Note, in turn, the use of a generative scheme for producing number names contains the seeds of the multiplication operation.

Another principle that preschool children seem not to understand is the principle that the list of number words is indeed unending. Their appreciation of this principle, like their appreciation of the generative principles for producing ever more numerlogs seems to depend upon experience with counting ever larger sets and the appreciation of the generative power of a base system. This principle, of course, leads on to a beginning under-

[1]Gelman and Gallistel (1978) coined the term numerlog to deal with the fact that the verbal tags used in a count need not to be the number words. The latter are one kind of a set of numerlogs.

standing of the infinity of the numbers, a concept that preschool children seem not to have (Evans and Gelman, 1982).

Thirdly, the application of counting procedures in the course of solving subtraction can lead to the recognition that there are 'numbers' other than those that may be arrived at in the course of ordinary counting, for example, zero (Evans, 1983). Preschool children seem to recognize only those numbers that they can arrive at by a counting procedure.

The findings presented in this paper underscore the need for theories of cognitive development to distinguish between implicit principles of knowledge and skills that derive from and/or are related to these principles. It will not do to base accounts of knowledge on a requirement that skills which reflect that knowledge be executed without error. Errors due to performance demands are to be expected.

References

Bullock, M., and Gelman, R. (1977) Numerical reasoning in young children: The ordering principle. *Child Devel., 48*, 427–434.

Clark, H. H., and Clark, E. V. (1977) *Psychology and Language: An Introduction to Psycholinguistics.* New York, Harcourt Brace Jovanovich, Inc.

Evans, D. (1983) *Understanding Infinity and Zero in the Early School Years.* Unpublished doctoral dissertation, University of Pennsylvania.

Evans, D. and Gelman, R. (1982) *Understanding Infinity: A Beginning Inquiry.* Unpublished manuscript, University of Pennsylvania.

Flavell, J. H., and Wellman, H. M. (1977) Metamemory. In R. V. Kail, Jr., and J. W. Hagen (Eds.), *Perspectives on the Development of Memory and Cognition.* Hillsdale, NJ, Erlbaum.

Fuson, K. C. and Mierkiewicz, D. B. (1980) *A Detailed Analysis of the Act of Counting.* Paper presented in April at American Educational Research Association, Boston.

Fuson, K. C., and Richards, J. (1979) *Children's Construction of the Counting Numbers: From a Spew to a Bidirectional Chain.* Northwestern University, July, 1979; unpublished manuscript.

Gast, H. (1957) Der Umgang mit Zahlen und Zahlgebilden in der frühren Kindeit. *Zeitschrift für Psychologie, 161*, 1–90.

Gelman, R. (1977) How young children reason about small numbers. In N. J. Castellan, D. B. Pisoni and G. R. Potts (eds.), *Cognitive Theory*, Vol. 2. Hillsdale, New Jersey: Erlbaum.

Gelman, R., and Gallistel, C. R. (1978) *The Child's Understanding of Number.* Cambridge, Mass., Harvard University Press.

Gleitman, L. R., Gleitman, H., and Shipley, E. F. (1972) The emergence of the child as grammarian. *Cog., 1*, 137–164.

Greenô, J. G., Riley, M. S., and Gelman, R. (1982) *Young Children's Counting and Understanding of Principles.* Unpublished manuscript, University of Pittsburgh and University of Pennsylvania.

Klahr, D., and Wallace, J. G. (1976) *Cognitive Development, an Information Processing View.* Hillsdale, NJ, Erlbaum.

Mandler, G., and Shebo, B. J. (In press) Subitizing: An analysis of its component processes. *J. exper. Psychol.: Gen.*

Mierkiewicz, D. B., and Siegler, R. S. (1981) *Preschoolers' Ability to Recognize Counting Errors.* Presented at the Spring meeting of the Society for Research on Child Development, Boston.

Saxe, G. B., and Moylan, T. (1982) *Developing Systems for Number Representation and Arithmetic Operations among the Oksapmin.* Paper presented in March at the American Education Research Association, New York.

Siegler, R. S. (1979) What young children do know. *Contemp. Psychol., 24*, 613–615.

Sternberg, R. J. (1980) Capacity of young children. *Sci., 208*, 47–48.

Wilkinson, A. C. (1981) *Children's Partial Knowledge of Counting.* Unpublished manuscript, University of Wisconsin, Madison.

Zaslavsky, C. (1973) *Africa Counts.* Boston, Prindle, Weber and Schmidt.

Résumé

Des enfants de 3 à 5 ans ont passé une parmi quatre expériences de comptage. On suppose que les exigences de la performance peuvent masquer une connaissance implicite des principes de comptage. Trois expériences testent la capacité de l'enfant à détecter les erreurs d'une poupée dans l'application des principes de comptage 'un à un', 'd'ordre stable' et 'cardinal'. Dans la quatrième expérience les enfants doivent compter dans différentes conditions pour faire varier les performances. Dans les expériences de détection d'erreurs où les enfants n'ont pas à compter, nous prédisons une bonne performance même avec des séries qui dépassent les capacités de comptage précis de l'enfant. La réussite des enfants renforce l'idée que les erreurs de comptage—au moins avec les séries au delà de 20, reflètent des exigences de la performance de comptage. Lorsque, comme dans la dernière expérience, les enfants comptent eux-mêmes la réussite est influencée par la taille de la série. Certaines variations des conditions influent, la plus difficile étant celle où les enfants doivent compter les objets tri-dimensionnels placés sous une couverture de plexiglass. Dans cette condition il y a interférence avec la tendance de l'enfant à pointer ou toucher les objets pour garder séparés les items déjà comptés des items non-comptés.